Fuzzy Set Theoretical Analysis of the Membership Values on the RGB Color Triangle

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***Abstract*—The present study considers a fuzzy color system in which three membership functions are constructed on the RGB color triangle. This system can process a fuzzy input (as the membership values of subjects) to an RGB system and output the center of gravity of three weights associated with respective grades. Three membership functions are applied to the RGB color triangle relationship. By treating three membership functions of redness, greenness, and blueness on the RGB color triangle, an average color value can be easily obtained as the center of gravity of the output fuzzy set. In the present paper, the differences among fuzzy input, inference output, and chromaticity are described, and the relationship between the centers of fuzzy inputs and inference outputs for fuzzy inputs are shown on the chromaticity diagram.**

I. Introduction

A technique for obtaining expressions of the RGB color triangle using the fuzzy set theoretical method has been reported [5] and improved [6]. In the previous study, the relationship between input fuzzy sets with a plateau on the RGB triangle and fuzzy inputs of conical membership functions was examined. The RGB color triangle (plane) represents the hue and saturation of a color [7]. The six fundamental colors and white can be represented on the same color triangle (See Fig.1). Vague colors on the RGB color triangle and chromaticity diagram were clarified. In the present study, the membership value on the RGB triangular system are examined to determine the average color value as the center of gravity of the attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

II. RGB Color Triangle

Additive color mixing occurs when two or three beams of differently colored light combine. It has been found that mixing just three additive primary colors, red, green and blue, can produce the majority of colors. In general, a color can be described by certain quantities, called the tristimulus values, r for the red component, g for the green component, and b for the blue component, as follows:

$$\text{color} = r + g + b \qquad (1)$$

This is called the RGB color model. This concept allows colors to be represented by a planar diagram. The first step is to draw the red, green and blue components as the vertices of a color triangle, as in Fig. 1. The coordinates on the plane of the color triangle can specify various colors. The location given by the coordinates corresponds to the amounts of r, g and b that make up the color. The coordinates specifying the center of the color triangle represent the case in which the three primary colors are mixed in equal proportion and indicate the color white. Such representations are called chromaticity diagrams. The diagram represents hue and saturation but not lightness [7]. On the RGB color triangle, the percentages of redness, greenness, and blueness, where the total of the three attributes is equivalent to 100% (as shown in Table I), specify a color.

III. Fuzzy System

The previous study [5], [6] considered a system of the three primary colors, red, green, and blue (RGB), presented on an RGB color triangle. As Fig. 1 shows, blue, cyan, green, yellow, red, magenta, and white are abbreviated as *B*, *C*, *G*, *Y*, *R*, *M*, and *W*, respectively. Six fundamental color coordinates, e.g., (r_1, g_1, b_1), (r_6, g_6, b_6), (r_{11}, g_{11}, b_{11}), …, were selected, where r_n, g_n, and b_n are the red, green, and blue components, respectively, of the n^{th} color.

Figure 2 corresponds to the schematic diagram shown in Fig. 1. The color names in Fig. 2 are No.1: blue, No.6: cyan, No.11: green, No.46: magenta, No.51: yellow, and No.66: red.

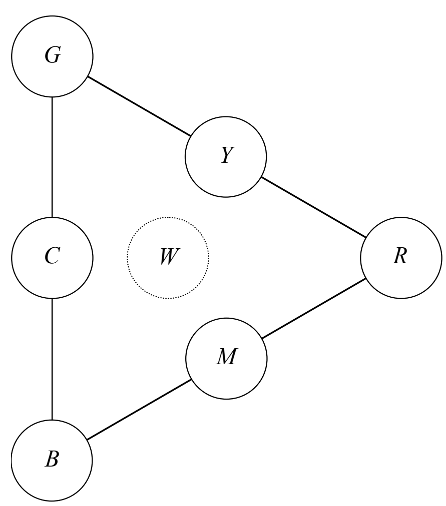

Fig. 1. A color triangle: A point on the plane of the triangular system represents the hue and saturation of a color spaces.

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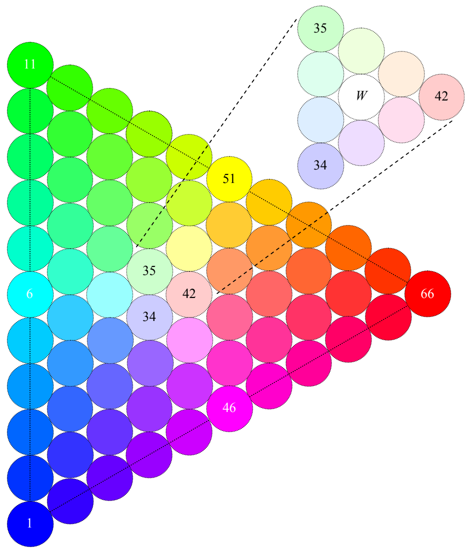

Fig. 2. Sixty-six crisp color inputs (*fundamental type*) and white with six neighboring colors (*detail type*) on the RGB color triangle.

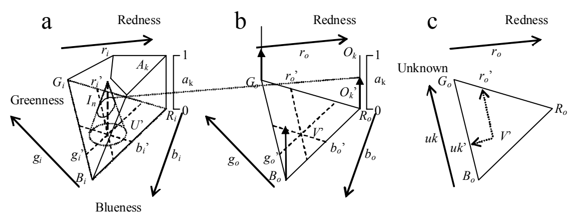

Fig. 3. Fuzzy system using the membership function of input fuzzy sets A_k, output crisp sets O_k and conical fuzzy input I_n on the RGB color triangle.

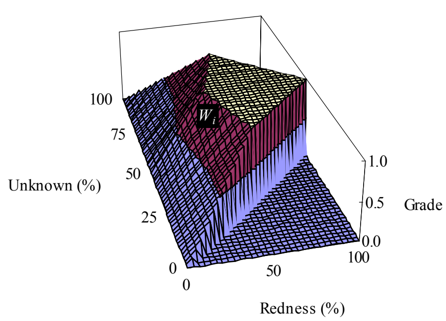

Fig. 4. The membership function μ_1 (r_i, uk) of input fuzzy set A_1 (*redness*) on the RGB color triangle. This is corresponding to Fig.3a.

White is surrounded by six neighboring colors, as shown in the detail inset, and these seven colors are surrounded by No.34, No.35, and No.42. We also try to examine the detail type which is extended from 66 colors in the fundamental type to 496 colors (including to white).

Figure 3 illustrates input fuzzy set, fuzzy input, output crisp set, and fuzzy output on the RGB color triangle, and crisp output on the graphical plane. The fuzzy rules are as follows (See Figs. 3 and 6):

R^1 : IF U is A_1 THEN V is O_1 (2)

R^2 : IF U is A_2 THEN V is O_2 (3)

R^3 : IF U is A_3 THEN V is O_3 (4)

Rule R^k : if U is A_k, then V is O_k ($k = 1, 2, 3$), where k is the rule number, A_k is a fuzzy set of inputs, O_k is a crisp set of outputs, $U = (r_i, g_i, b_i)$ are input parameters (variable), and $V = (r_o, g_o, b_o)$ are output parameters. Here, U and V are fixed to these RGB parameters. A fuzzy set A_k of inputs shows a triangular pyramid-like shape at corner points R_i, G_i, and B_i, and a crisp set O_k of outputs of rule R^k is shown at corner points R_o, G_o, or B_o (a fuzzy set O_k' indicated by vertical arrows in Fig. 3b) on the color triangle, and the output is O_k if the input is A_k.

The fuzzy inference method is as follows. Let the inputs be $r_i = r_i'$, $g_i = g_i'$, and $b_i = b_i'$.

1) The input of rule R^k, grade $\alpha_k = A_k(U')$, where $k = 1, 2, 3$.

2) The output of rule R^k, output crisp set is shown as a vertical post.

3) $O_k' = \alpha_k O_k$, where O_k' is fuzzy sets (as vertical allows) and O_k is crisp sets (as vertical posts) in Fig. 3b. The complete inference results O' of rules R^1, R^2, and R^3.

$O' = \alpha_1 O_1 \cup \alpha_2 O_2 \cup \alpha_3 O_3 = O_1' \cup O_2' \cup O_3'$ (5)

The output parameter, $V' = (r_o', g_o', b_o')$, corresponds to the coordinates of the central axis of the membership function of O'. In addition, in Fig. 3c, $V' = (r_o', uk')$ corresponds to a coordinates of the graphical system, where uk' (on the vertical axis) is calculated from g_o' and b_o'. uk' shows a value (as distance from B) on the line B-G.

An input fuzzy set A_1 of *redness* can be characterized by the following membership function:

$\mu_1 (r_i, uk) = r_i\, s;\ r_i < 1/s$ (6)

$\mu_1 (r_i, uk) = 1;\ r_i \geq 1/s$ (7)

, where s is slope of projection and s ranges from 0.02 to 0.03 (See Fig.6). The limitations of uk are as follows:

$50 \geq uk \geq r_i /2$ (8)

$50 < uk \leq -(r_i /2) +100$ (9)

The membership functions of *greenness* and *blueness* are also described by similar equations.

Table I shows the membership value $\mu_k (r_i', g_i', b_i')$ of input fuzzy set A_k on the RGB color triangle. $\mu_k (r_i', g_i', b_i')$ is equal to $\mu_k (r_i', uk')$. The membership function μ_k was based on the values of seven colors (R, Y, G, C, B, M, and W).

In Fig.4, the shape of membership function is shown by including to W_i (white). Top of the plateau is shown as diamond-like shape in this case. See also Table I and Fig.6.

Figure 5a (left) illustrates twenty-one fuzzy inputs (I_1–I_6, I_{12}–I_{16}, I_{22}–I_{25}, I_{31}–I_{33}, I_{39}–I_{40}, and I_{46}) on the RGB color triangle as a triangle with color names (B, C, and M). The fuzzy inputs are formed by conical membership functions, and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles.

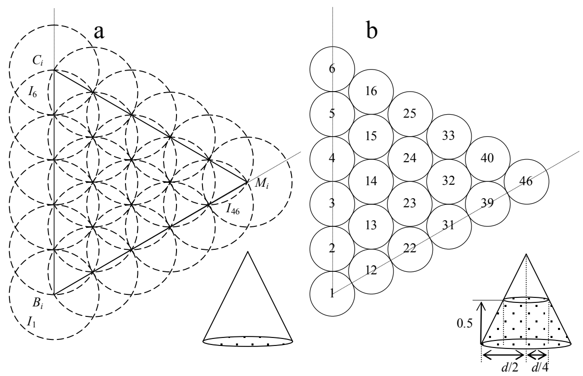

Fig. 5. Fuzzy inputs on part of the RGB color triangle and top areas of 0.5 level-sets indicated by number. The diameter (d = 23.0%) of the basal plane (*circle*) of the cone indicated vagueness.

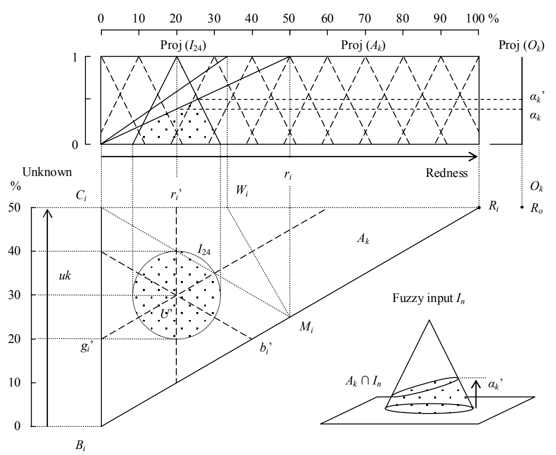

Fig. 6. Membership functions of input fuzzy sets A_k on half of the RGB color triangle and one of sixty-six conical fuzzy inputs (*vague colors*).

TABLE I
MEMBERSHIP VALUE OF INPUT FUZZY SET A ON THE RGB COLOR TRIANGLE

Color	Color coordinate			Membership value μ_k		
	r_i'	g_i'	b_i'	k=1	k=2	k=3
B_i	0.0	0.0	100.0	0.00	0.00	1.00
C_i	0.0	50.0	50.0	0.00	1.00	1.00
G_i	0.0	100.0	0.0	0.00	1.00	0.00
M_i	50.0	0.0	50.0	1.00	0.00	1.00
Y_i	50.0	50.0	0.0	1.00	1.00	0.00
R_i	100.0	0.0	0.0	1.00	0.00	0.00
W_i	33.3	33.3	33.3	1.00	1.00	1.00

TABLE II
NUMBER OF SUBJECTS IN THE EXPERIMENT

No. of subjects	Male	Female	Age
66	60	6	10-50

Figure 5b (right) shows the arrangement of numbers corresponding to the conical membership functions of Fig. 5a, and the numbers are shown inside circles representing the top of the 0.5 level-set (bottom-right). The color names are No.1: blue, No.6: cyan, and No.46: magenta.

Figure 6 illustrates half of the RGB color triangle as a base of input fuzzy set A_k and one of the sixty-six conical fuzzy inputs (I_1–I_{66}) on the RGB color triangle. For k = 1 (as redness), sharp slant line (s = 0.03) shows a projection of line between C_i with membership value $\mu_k = 0$ and W_i with $\mu_k = 1$ and gentle slope line (s = 0.02) shows a projection of line between B_i with value $\mu_k = 0$ and M_i with value $\mu_k = 1$ (or between G_i with $\mu_k = 0$ and Y_i with $\mu_k = 1$ on the blind side). See also Table I and Fig.4. The triangular membership function Proj(I_{24}) on the redness axis is one of eleven projections of the sixty-six fuzzy inputs (I_1–I_{66}) by the rays from the lower part, and the triangular membership function Proj(I_n) on the unknown axis is not used in the present study.

The intersection of input fuzzy set A_k for fuzzy input I_n is $A_k \cap I_n$. (See the dotted area at the bottom-right of Fig.6.) Grade α_k' = height ($A_k \cap I_n$). If the input is crisp, α_k' becomesα_k. R_o is the new red as output. Proj(O_k) is a projection of an output crisp set at the corner point R_o (See Fig. 3b).

What happens if a vague color is input into the RGB system? The system considered in this study can translate input data U of vague color to output data V of simple color on the RGB color triangle. The fuzzy input on the RGB is constructed by the center U' = (r_i', g_i', b_i') = (0, 0, 100) in % and the diameter d of the basal plane (circle) of the cone indicated vagueness.

IV. EXPERIMENTAL METHODS

For the experiment, 15 (in Table II) undergraduate students, graduate students, and participants in a university festival volunteered to participate as subjects for this study. The subjects sat in a chair and were requested to watch a display continuously.

First, the subjects picked up red-relevant colors, green -relevant colors, blue-relevant colors and white-relevant colors from 496 colors in detail type. Three groups of the RGB crisp sets are determined. Using the ensemble average as the RGB fuzzy sets, then, the normalized membership values of 15 subjects are computed.

Second, the subjects picked up red-relevant colors, yellow -relevant colors, green-relevant colors, cyan-relevant colors, blue-relevant colors, and magenta-relevant colors from 66 colors in fundamental type (See Fig.2). Six groups of the RYGCBM crisp sets are determined in the same manner as 496 colors (detail type).

Third, using a graphical user interface (GUI) for the questionnaire, 51 subjects compared the differences between a target color (e.g. red) and neighboring colors of 66 colors in fundamental type (See Fig.2). Three groups of the RGB fuzzy sets are determined. The experiments were performed in an isolated area in order to restrict visual cues with regard to the display.

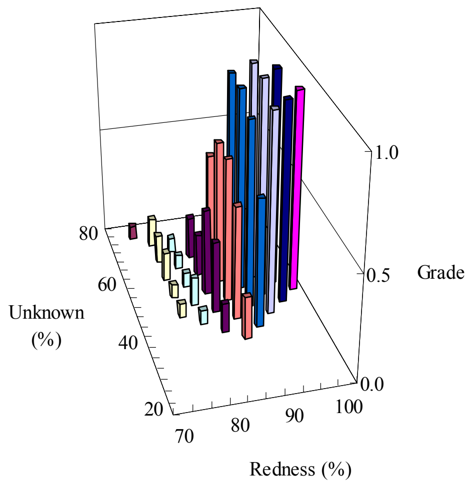

Fig. 7. Membership values of thirty-three red-relevant colors on the coordinates (r_i, uk).

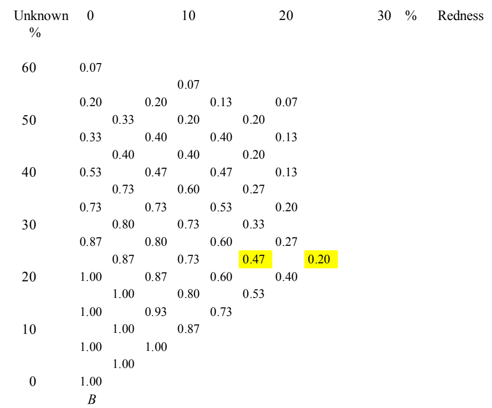

Fig. 10. Membership values of fifty-four blue-relevant colors (blue and neighboring colors) on the coordinates (r_i, uk).

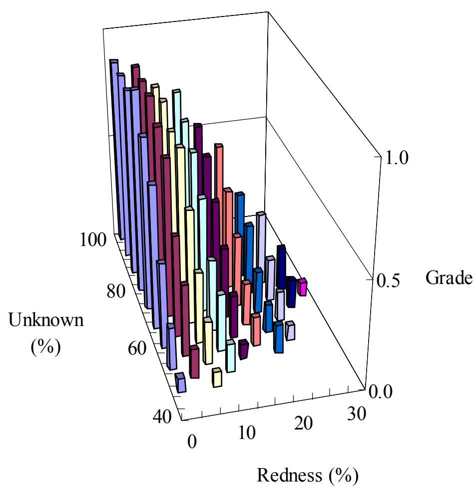

Fig. 8. Membership values of fifty-five green-relevant colors on the coordinates (r_i, uk).

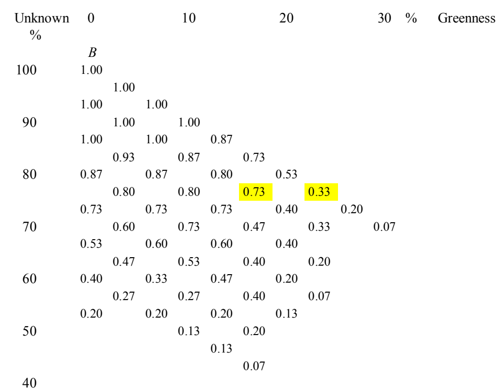

Fig. 11. Membership values of fifty-four blue-relevant colors (blue and neighboring colors) on the coordinates (g_i, uk).

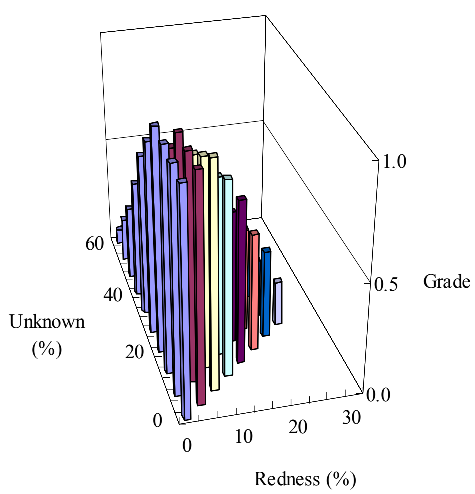

Fig. 9. Membership values of fifty-four blue-relevant colors on the coordinates (r_i, uk).

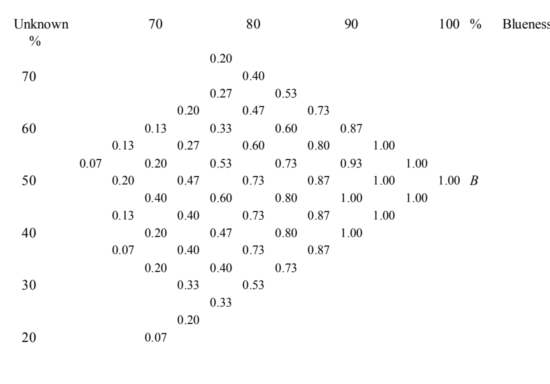

Fig. 12. Membership values of fifty-four blue-relevant colors (blue and neighboring colors) on the coordinates (b_i, uk).

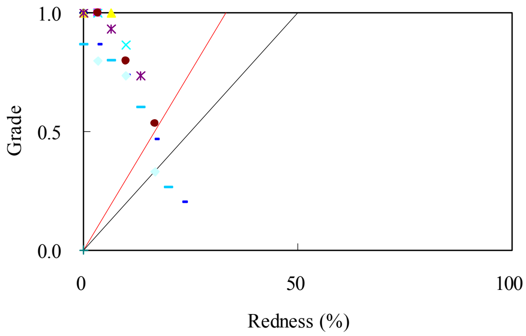

Fig. 13. Projections of membership values of blue-relevant colors and projections of input fuzzy set of redness.

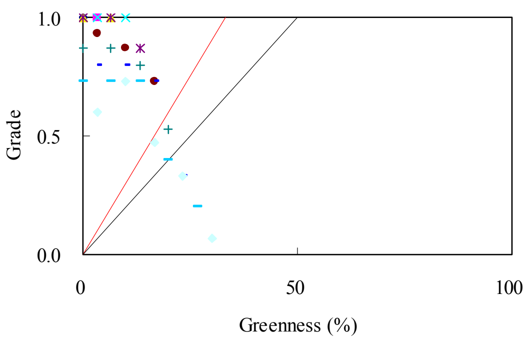

Fig. 14. Projections of membership values of blue-relevant colors and projections of input fuzzy set of greenness.

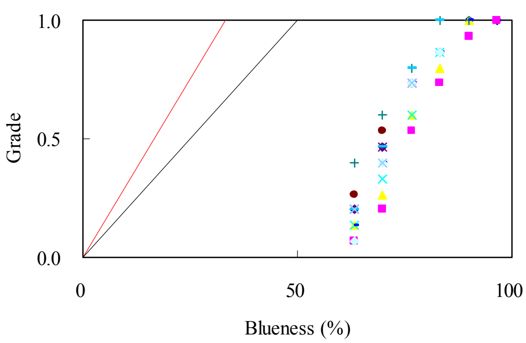

Fig. 15. Projections of membership values of blue-relevant colors and projections of input fuzzy set of blueness.

Equipment

An HP Compaq 14.1” Liquid Crystal Display was used to present the stimulus pattern. The display resolution was 1024×768 pixels/60 Hz.

V. Experimental Results and Discussion

Figures 7-9 show the experimental results for red-relevant colors, green-relevant colors, and blue-relevant colors on the coordinates (r_i’, uk) in detail type. The membership values of 15 subjects are combined.

Figure 10 is corresponding to Fig.6 (lower part). This figure shows a top view of Fig.9. Although the directions (horizontal axes) are different, Figs.10-12 show the same data for blue-relevant colors. Unknown axes are equal to the parts of lines *B*–*G*, *G*–*R*, and *R*–*B* (the edges of RGB color triangle), respectively.

Figures 13-15 are corresponding to projections in Fig.6 (upper part). Gentle slopes and sharp slopes are $s = 0.02$ and $s = 0.03$, respectively. See Eqs.6–9. The marked data seem to a half of triangle-like projection of conical membership function. It assumes that the fuzzy inputs on the RGB are constructed by the centers (r_i’, g_i’, b_i’). The center (R_i) of red-relevant colors is (100, 0, 0), the center (G_i) of green-relevant colors is (0, 100, 0), and the center (B_i) of blue-relevant colors is (0, 0, 100), which assume to fuzzy input I_n in Fig.5a. The coordinates of centers and that of inference outputs are examined.

Red-relevant colors, green-relevant colors, and blue -relevant colors have a plateau peak in Figs.7-9. White -relevant colors are gathered about one coordinate (33.3, 50) in detail type. Yellow-relevant colors are gathered round coordinate (50, 75). Cyan-relevant colors are gathered round coordinate (0, 50). However, magenta-relevant colors have not sharp peak at coordinate (50, 25). These four-relevant colors are not shown herein.

The calculation of intersections between membership values (divided into red, green, and blue) and projection of membership function of input fuzzy set are performed. In the conditions ($uk \leq 25\%$ or $uk \geq 75\%$) of Figs.10 and 11, gentle slope $s = 0.02$ is used. See Eqs.6–9. Shaded numbers of Figs.10 (for redness) and 11 (for greenness) where the intersections are the highest in Figs.13 (for redness) and 14 (for greenness) are used in the calculations. In Fig.15 (for blueness), the highest value is equal to 1.0, because a fuzzy set of blue-relevant colors is a subset of input fuzzy set of blueness.

Figure 16 illustrates a relationship between the vertical value uk and the redness value r_o. Filled circles indicate outputs for fuzzy inputs of colors, open circles indicate crisp inputs of colors. The outputs (filled circles) for fuzzy inputs are grouped at the center of the RGB triangle. The open and filled circles are clearly different in this case. These inference results for fuzzy 496 colors (R_{f1}, G_{f1}, B_{f1}) and fuzzy 66 colors (R_{f2}, G_{f2}, B_{f2}) are similar. $W < R_{f2} \leq R_{f1} < R$ for redness, $W < G_{f2} \leq G_{f1} < G$ for greenness, $W < B_{f2} \leq B_{f1} < B$ for blueness. The experimental results for GUI are not shown in this paper.

The chromaticity coordinates are denoted by r_o’, g_o’, b_o’ and *x*, *y*, *z*. The transformation from *R*, *G*, and *B* to X_i, Y_i, and Z_i can be shown as follows [2], [3]:

$$X_i = 2.77\, r_o' + 1.75\, g_o' + 1.13\, b_o' \quad (10)$$

$$Y_i = 1.00\, r_o' + 4.59\, g_o' + 0.06\, b_o' \quad (11)$$

$$Z_i = 0.00\, r_o' + 0.06\, g_o' + 5.59\, b_o' \quad (12)$$

The general definitions of the chromaticies *x*, *y*, *z* [1], [4] are:

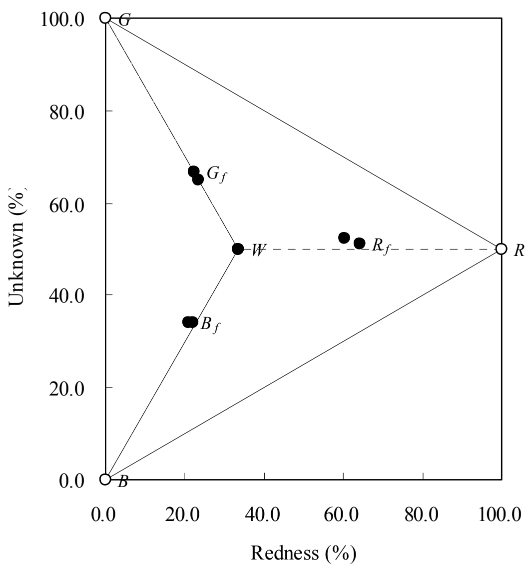

Fig. 16. Crisp inputs (*open circles*) and inference outputs (*filled circles*) for fuzzy inputs on the graphical plane

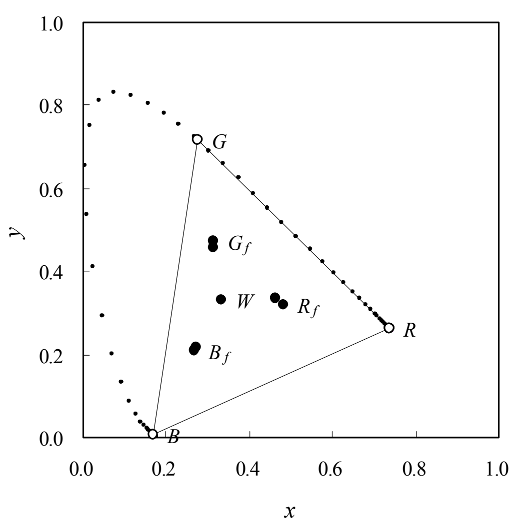

Fig. 17. Crisp inputs (*open circles*) and inference outputs (*filled circles*) for fuzzy inputs on the chromaticity diagram. Dotted points are CIE1931 chromaticties

$$x = X_i / (X_i + Y_i + Z_i) \quad (13)$$

$$y = Y_i / (X_i + Y_i + Z_i) \quad (14)$$

$$z = Z_i / (X_i + Y_i + Z_i) \quad (15)$$

, where $x + y + z = 1$.

Figure 17 illustrates the differences between crisp inputs and inference outputs for fuzzy inputs. Only the three fundamental colors (filled circles) computed in this study show the movements in direction from coordinates (x, y) for crisp input to those for fuzzy input. The direction indicates toward white W (filled circles), for example, R_f lies midway between R and W. The output (x, y) for fuzzy input is dislocated from the center of fuzzy input (vague colors). Vague colors move toward the direction of white. It implies that the distance from the center of fuzzy inputs to inference outputs for fuzzy inputs is also longer, if vagueness becomes larger. Dotted points show CIE1931 chromaticties (x, y), wave-length λ = 360 - 830 nm [4].

Vague color inputs to the RGB (Figs.7-9), the system outputs crisp color in the RGB, and also outputs crisp color on the graphical plane (Fig.16), or on the chromaticity diagram (Fig.17). These inference results for fuzzy 496 colors and fuzzy 66 colors are similar.

VI. Conclusion

The present paper examined how vagueness is presented on the RGB color triangle using two methods (partition and semantic differential methods) and performed fuzzy set theoretical analysis. First, the subjects are asked where partitions of fundamental colors are on the triangle. The subjects divide RGB color triangle in two areas with a partition. The number of neighboring colors is different for each subject. It considered that the data of each subject is a crisp set, the ensemble average of all the data is a fuzzy set. Second, the subjects are also asked the difference between fundamental color (as a target color) and neighboring colors (as a sample color) using semantic differential method. Each data and the ensemble average of those data are fuzzy sets. The results of two experiments show a similar trend. Using the fuzzy inference for RGB data (as a fuzzy set), it is found that these results move to white direction as a center of RGB color triangle. For instance the inference results as if the red-relevant colors are equal to *vermilion* are appeared in the present study.

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