原著論文

Fuzzy Set Theoretical Approach to the RGB Triangular System

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The present study considers a fuzzy color system in which three membership functions are constructed on the RGB color triangle. This system can process a fuzzy input to an RGB system and output the center of gravity of three weights associated with respective grades. Three membership functions are applied to the RGB color triangle relationship. By treating three membership functions of redness, greenness, and blueness on the RGB color triangle, a target color can be easily obtained as the center of gravity of the output fuzzy set. In the present paper, the differences among fuzzy input, inference output, and chromaticity are described, and the relationship between inference outputs for crisp inputs and inference outputs for fuzzy inputs are shown on the chromaticity diagram.

Keywords：Fuzzy set theoretical approach, Three additive primary color, RGB color triangle, Vague color, RGB membership function, Conical membership function, Chromaticity diagram

1．Introduction

Additive color mixing occurs when two or three beams of differently colored light combine. It has been found that mixing just three additive primary colors, red, green and blue, can produce the majority of colors. In general, a color can be described by certain quantities, called the tristimulus values, *r* for the red component, *g* for the green component, and *b* for the blue component, as follows:

$$\text{color} = r + g + b \qquad (1)$$

This is called the RGB color model. This concept allows colors to be represented by a planar diagram. The first step is to draw the red, green and blue components as the vertices of a color triangle, as in Fig. 1. The coordinates on the plane of the color triangle can specify various colors. The location given by the coordinates corresponds to the amounts of *r*, *g* and *b* that make up the color. The coordinates specifying the center of the color triangle represent the case in which the three primary colors are mixed in equal proportion and indicate the color white. Such representations are called chromaticity diagrams. The diagram represents hue and saturation but not lightness [9]. On the RGB color triangle, the percentages of redness, greenness, and blueness, where the total of the three attributes is equivalent to 100% (as shown in Tables 2 and 3), specify a color.

In the Natural Color System (NCS), a method similar to the fuzzy set theoretical method for obtaining hue expressions with vagueness has been reported by Sivik [4]. Using the fuzzy set theoretical method, a technique for acquiring tone expressions with vagueness on the NCS color triangle has been investigated by Sugano [5], [7]. In a recent study, the triangular membership functions of achromatic colors and conical membership functions of chromatic colors were used as vagueness [5], [7], which caused a gathering effect toward the center of the NCS tone triangle. In this previous study, fuzzy achromatic colors of triangular membership functions and fuzzy modified achromatic colors of conical membership functions were used on the NCS color triangle in a manner corresponding to the HLS (hue, lightness, and saturation) tone plane consisting of lightness and saturation. The vagueness effects of achromatic colors and modified achromatic colors (e.g., reddish, yellowish, greenish, and bluish achromatic colors) have been clarified [6], [8].

However, a technique for obtaining expressions of the RGB color triangle using the fuzzy set theoretical

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method has not been reported. In the present study, the relationship between input fuzzy sets with a plateau on the RGB triangle and fuzzy inputs of conical membership functions is examined. The RGB color triangle (plane) represents the hue and saturation of a color [9]. The six fundamental colors and white can be represented on the same color triangle (See Fig.1). Vague colors on the RGB color triangle and chromaticity diagram are clarified. Such a system will help us to determine the average color value as the center of gravity of the attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

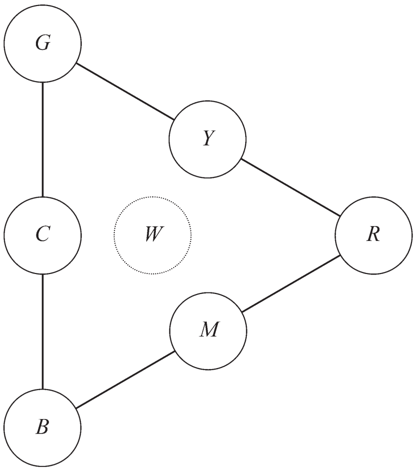

Fig.1 A color triangle. A point on the plane of the triangular system represents the hue and saturation of a color

2. Methods

The present study considers a system of the three primary colors, red, green, and blue (RGB), presented on an RGB color triangle. As Fig. 1 shows, blue, cyan, green, yellow, red, magenta, and white are abbreviated as B, C, G, Y, R, M, and W, respectively. Six fundamental color coordinates, e.g., (r_1, g_1, b_1), (r_6, g_6, b_6), (r_{11}, g_{11}, b_{11}), ⋯, were selected, where r_n, g_n, and b_n are the red, green, and blue components, respectively, of the n^{th} color.

Figure 2 corresponds to the schematic diagram shown in Fig. 1. The color names in Fig. 2 are No.1 : blue, No.6 : cyan, No.11 : green, No.46: magenta,

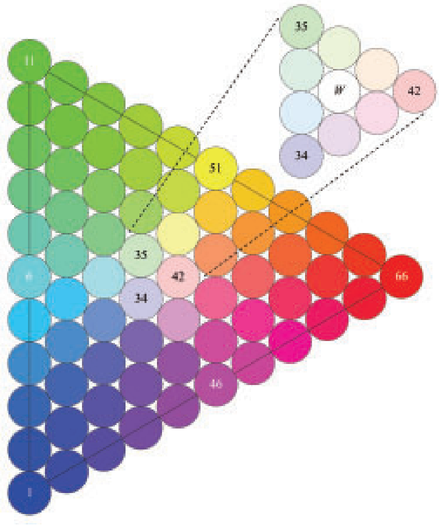

Fig.2 Sixty-six crisp color inputs and white with six neighboring colors (*detail*) on the RGB color triangle

No.51 : yellow, and No.66 : red. White is surrounded by six neighboring colors, as shown in the detail inset, and these seven colors are surrounded by No.34, No.35, and No.42.

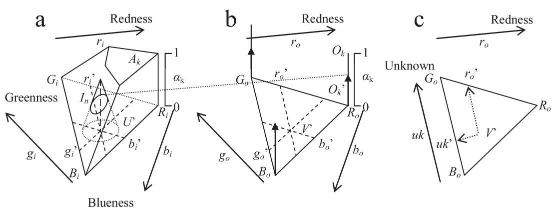

Fig.3 Fuzzy system using the membership function of input fuzzy sets A_k, output crisp sets O_k and conical fuzzy input I_n on the RGB color triangle

Figure 3 illustrates input fuzzy set, fuzzy input, output crisp set, and fuzzy output on the RGB color triangle, and crisp output on the graphical plane. The fuzzy rules are as follows (See Figs. 3 and 6) :

$$R^1 : \text{IF } U \text{ is } A_1 \quad \text{THEN } V \text{ is } O_1 \tag{2}$$

$$R^2 : \text{IF } U \text{ is } A_2 \quad \text{THEN } V \text{ is } O_2 \tag{3}$$

$$R^3 : \text{IF } U \text{ is } A_3 \quad \text{THEN } V \text{ is } O_3 \tag{4}$$

Rule R^k : if U is A_k, then V is O_k $(k = 1, 2, 3)$, where k is the rule number, A_k is a fuzzy set of inputs, O_k is a crisp set of outputs, $U=(r_i, g_i, b_i)$ are input parameters (variable), and $V=(r_o, g_o, b_o)$ are output parameters. Here, U and V are fixed to these RGB parameters. A fuzzy set A_k of inputs shows a triangular pyramid-like shape at corner points R_i, G_i, and B_i, and a crisp set O_k of outputs of rule R^k is shown at corner points R_o, G_o, or B_o (a fuzzy set O_k' indicated by vertical arrows in Fig. 3b) on the color triangle, and the output is O_k if the input is A_k.

The fuzzy inference method is as follows. Let the inputs be $r_i=r_i'$, $g_i=g_i'$, and $b_i=b_i'$.

1) The input of rule R^k, grade $a_k=A_k(U')$, where $k = 1, 2, 3$.
2) The output of rule R^k, output crisp set is shown as a vertical post.
3) $O_k' = a_k O_k$, where O_k' is fuzzy sets (as vertical allows) and O_k is crisp sets (as vertical posts) in Fig. 3b. The complete inference results O' of rules R^1, R^2, and R^3.

$$O' = \alpha_1 O_1 \cup \alpha_2 O_2 \cup \alpha_3 O_3 = O_1' \cup O_2' \cup O_3' \tag{5}$$

The output parameter, $V'=(r_o', g_o', b_o')$, corresponds to the coordinates of the central axis of the membership function of O'. In addition, in Fig. 3c, $V'=(r_o', uk')$ corresponds to a coordinates of the graphical system, where uk' (on the vertical axis) is calculated from g_o' and b_o'. uk' shows a value (as distance from B) on the line B-G.

An input fuzzy set A_1 of *redness* can be characterized by the following membership function :

$$\mu_1 (r_i, uk) = r_i s; \qquad r_i < 1/s \tag{6}$$

$$\mu_1 (r_i, uk) = 1; \qquad r_i \geq 1/s \tag{7}$$

where s is slope of projection and s ranges from 0.02 to 0.03 (See Fig.6). The limitations of uk are as follows :

$$50 \geq uk \geq r_i/2 \tag{8}$$

$$50 < uk \leq -(r_i/2) + 100 \tag{9}$$

The membership functions of *greenness* and *blueness* are also described by similar equations.

Table 1 shows the membership value $\mu_k (r_i', g_i', b_i')$ of input fuzzy set A_k on the RGB color triangle. $\mu_k (r_i', g_i', b_i')$ is equal to $\mu_k (r_i', u_k')$. The membership function μ_k was based on the values of seven colors (R, Y, G, C, B, M, and W).

In Fig.4, the shape of membership function is shown by including to W_i (white). Top of the plateau is shown as diamond-like shape in this case. See also Table 1 and Fig.6.

Figure 5a (left) illustrates twenty-one fuzzy inputs (I_1–I_6, I_{12}–I_{16}, I_{22}–I_{25}, I_{31}–I_{33}, I_{39}–I_{40}, and I_{46}) on the RGB color triangle as a triangle with color names (B, C, and, M). The fuzzy inputs are formed by conical membership functions, and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles. Figure 5b (right) shows the arrangement of numbers corresponding to the conical membership functions of Fig. 5a, and the numbers are shown inside circles representing the top of the 0.5 level-set (bottom-right). The color names are No.1: blue, No.6 : cyan, and No.46 : magenta.

Figure 6 illustrates half of the RGB color triangle as a base of input fuzzy set A_k and one of the sixty-six conical fuzzy inputs (I_1–I_{66}) on the RGB color triangle. For $k=1$ (as redness), sharp slant line ($s=0.03$)

Table 1 Membership value $\mu_k (r_i', g_i', b_i')$ of input fuzzy set A_k on the RGB color triangle

Color	Color coordinate			Membership value $\mu_k (r_i', g_i', b_i')$			Graphic coordinate	
	r_i'	g_i'	b_i'	k=1	k=2	k=3	r_i'	uk'
B_i	0.0	0.0	100.0	0.00	0.00	1.00	0.0	0.0
C_i	0.0	50.0	50.0	0.00	1.00	1.00	0.0	50.0
G_i	0.0	100.0	0.0	0.00	1.00	0.00	0.0	100.0
M_i	50.0	0.0	50.0	1.00	0.00	1.00	50.0	25.0
Y_i	50.0	50.0	0.0	1.00	1.00	0.00	50.0	75.0
R_i	100.0	0.0	0.0	1.00	0.00	0.00	100.0	50.0
W_i	33.3	33.3	33.3	1.00	1.00	1.00	33.3	50.0

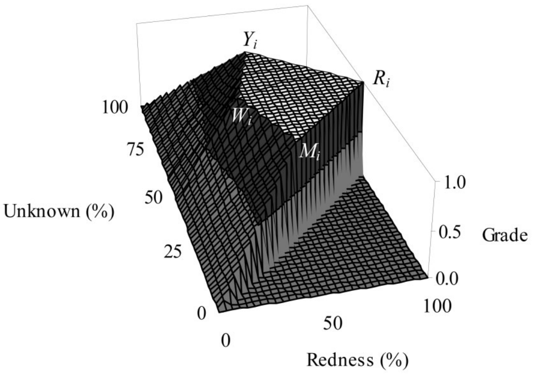

Fig.4 The membership function $\mu_1(r_i, uk)$ of input fuzzy set A_1(*redness*) on the RGB color triangle

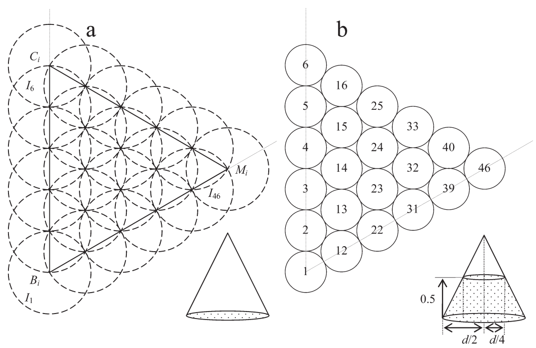

Fig.5 Fuzzy inputs on part of the RGB color triangle and top areas of 0.5 level- sets indicated by number. The diameter (d = 23.0%) of the basal plane (*circle*) of the cone indicated vagueness

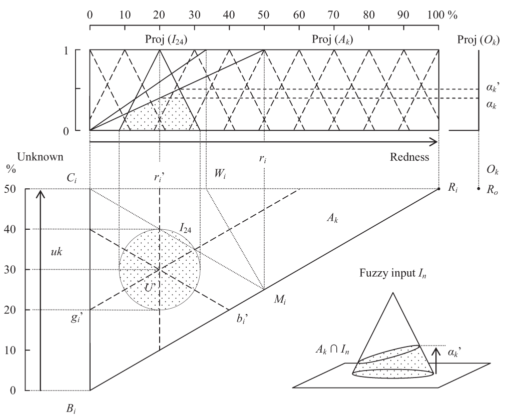

Fig.6 Membership functions of input fuzzy sets A_k on half of the RGB color triangle and one of sixty-six conical fuzzy inputs (*vague colors*)

shows a projection of line between C_i with membership value μ_k=0 and W_i with μ_k=1 and gentle slope line (s=0.02) shows a projection of line between B_i with value μ_k=0 and M_i with value μ_k=1 (or between G_i with μ_k=0 and Y_i with μ_k=1 on the blind side). See also Table 1 and Fig.4. The triangular membership function Proj(I_{24}) on the redness axis is one of eleven projections of the sixty-six fuzzy inputs (I_1-I_{66}) by the rays from the lower part, and the triangular membership function Proj(I_n) on the unknown axis is not used in the present study.

The intersection of input fuzzy set A_k for fuzzy input I_n is $A_k \cap I_n$. (See the dotted area at the bottom-right of Fig. 6.) Grade a_k' =height ($A_k \cap I_n$). If the input is crisp, a_k' becomes a_k. R_o is the new red as output. Proj(O_k) is a projection of an output crisp set at the corner point R_o (See Fig. 3b).

3. Results and Discussion

What happens if a vague color is input into the RGB system? The system considered in the present study can translate input data U of a vague color to output data V of a simple color on the RGB color triangle. The fuzzy input (No.24) on the RGB is made up of the center U' =(r_i' , g_i' , b_i')=(20, 20, 60) in % and the diameter d=23.0% of the basal plane (circle) of the cone indicated vagueness.

Table 2 shows the fuzzy inference results for sixty-six crisp inputs (r_i' , g_i' , b_i'). The grades (a_1, a_2, a_3), inference outputs (r_o' , g_o' , b_o'), and graphic output V' =(r_o' , uk') were calculated. The relationship between inputs, grades, and outputs is not proportional. The color names are No.1 : blue, No.6 : cyan, No.11 : green, No.46 : magenta, No.51 : yellow, and No.66 : red. The suffix of fuzzy input I indicates these numbers.

Table 3 shows the fuzzy inference results for sixty-six fuzzy inputs. The fuzzy inputs on the RGB are constructed by the centers (r_i' , g_i' , b_i') in the same manner as the crisp inputs in Table 2 and the diameter (d = 23.0%) of the basal plane (circle) of the cone indicated vagueness (See Figs 5 and 6). The grades (a_1' , a_2' , a_3'), inference outputs (r_o' , g_o' , b_o'), and graphic output V' =(r_o' , uk')were calculated. The inputs and outputs are not proportional, which indicates nonlinear information processing in this fuzzy system. In the previous studies [5]-[8] the intersection of input

Table 2 Inference results for crisp inputs of RGB colors

No.	Crisp input			Grade for crisp input			Inference output			Graphic output	
	r_i'	g_i'	b_i'	α_1	α_2	α_3	r_o'	g_o'	b_o'	r_o'	uk'
1	0	0	100	0.00	0.00	1.00	0.0	0.0	100.0	0.0	0.0
2	0	10	90	0.00	0.20	1.00	0.0	16.7	83.3	0.0	16.7
3	0	20	80	0.00	0.40	1.00	0.0	28.6	71.4	0.0	28.6
4	0	30	70	0.00	0.60	1.00	0.0	37.5	62.5	0.0	37.5
5	0	40	60	0.00	0.80	1.00	0.0	44.4	55.6	0.0	44.4
6	0	50	50	0.00	1.00	1.00	0.0	50.0	50.0	0.0	50.0
7	0	60	40	0.00	1.00	0.80	0.0	55.6	44.4	0.0	55.6
8	0	70	30	0.00	1.00	0.60	0.0	62.5	37.5	0.0	62.5
9	0	80	20	0.00	1.00	0.40	0.0	71.4	28.6	0.0	71.4
10	0	90	10	0.00	1.00	0.20	0.0	83.3	16.7	0.0	83.3
11	0	100	0	0.00	1.00	0.00	0.0	100.0	0.0	0.0	100.0
12	10	0	90	0.20	0.00	1.00	16.7	0.0	83.3	16.7	8.3
13	10	10	80	0.22	0.22	1.00	15.3	15.3	69.4	15.3	22.9
14	10	20	70	0.24	0.45	1.00	14.2	26.6	59.2	14.2	33.7
15	10	30	60	0.27	0.69	1.00	13.8	35.2	51.0	13.8	42.1
16	10	40	50	0.29	0.90	1.00	13.2	41.1	45.7	13.2	47.7
17	10	50	40	0.29	1.00	0.90	13.2	45.7	41.1	13.2	52.3
18	10	60	30	0.27	1.00	0.69	13.8	51.0	35.2	13.8	57.9
19	10	70	20	0.24	1.00	0.45	14.2	59.2	26.6	14.2	66.3
20	10	80	10	0.22	1.00	0.22	15.3	69.4	15.3	15.3	77.1
21	10	90	0	0.20	1.00	0.00	16.7	83.3	0.0	16.7	91.7
22	20	0	80	0.40	0.00	1.00	28.6	0.0	71.4	28.6	14.3
23	20	10	70	0.45	0.24	1.00	26.6	14.2	59.2	26.6	27.5
24	20	20	60	0.50	0.50	1.00	25.0	25.0	50.0	25.0	37.5
25	20	30	50	0.55	0.77	1.00	23.7	33.2	43.1	23.7	45.0
26	20	40	40	0.60	1.00	1.00	23.1	38.5	38.5	23.1	50.0
27	20	50	30	0.55	1.00	0.77	23.7	43.1	33.2	23.7	55.0
28	20	60	20	0.50	1.00	0.50	25.0	50.0	25.0	25.0	62.5
29	20	70	10	0.45	1.00	0.24	26.6	59.2	14.2	26.6	72.5
30	20	80	0	0.40	1.00	0.00	28.6	71.4	0.0	28.6	85.7
31	30	0	70	0.60	0.00	1.00	37.5	0.0	62.5	37.5	18.8
32	30	10	60	0.69	0.27	1.00	35.2	13.8	51.0	35.2	31.4
33	30	20	50	0.77	0.55	1.00	33.2	23.7	43.1	33.2	40.3
34	30	30	40	0.86	0.86	1.00	31.6	31.6	36.8	31.6	47.4
35	30	40	30	0.86	1.00	0.86	31.6	36.8	31.6	31.6	52.6
36	30	50	20	0.77	1.00	0.55	33.2	43.1	23.7	33.2	59.7
37	30	60	10	0.69	1.00	0.27	35.2	51.0	13.8	35.2	68.6
38	30	70	0	0.60	1.00	0.00	37.5	62.5	0.0	37.5	81.3
39	40	0	60	0.80	0.00	1.00	44.4	0.0	55.6	44.4	22.2
40	40	10	50	0.90	0.29	1.00	41.1	13.2	45.7	41.1	33.8
41	40	20	40	1.00	0.60	1.00	38.5	23.1	38.5	38.5	42.3
42	40	30	30	1.00	0.86	0.86	36.8	31.6	31.6	36.8	50.0
43	40	40	20	1.00	1.00	0.60	38.5	38.5	23.1	38.5	57.7
44	40	50	10	0.90	1.00	0.29	41.1	45.7	13.2	41.1	66.2
45	40	60	0	0.80	1.00	0.00	44.4	55.6	0.0	44.4	77.8
46	50	0	50	1.00	0.00	1.00	50.0	0.0	50.0	50.0	25.0
47	50	10	40	1.00	0.29	0.90	45.7	13.2	41.1	45.7	36.1
48	50	20	30	1.00	0.55	0.77	43.1	23.7	33.2	43.1	45.3
49	50	30	20	1.00	0.77	0.55	43.1	33.2	23.7	43.1	54.7
50	50	40	10	1.00	0.90	0.29	45.7	41.1	13.2	45.7	63.9
51	50	50	0	1.00	1.00	0.00	50.0	50.0	0.0	50.0	75.0
52	60	0	40	1.00	0.00	0.80	55.6	0.0	44.4	55.6	27.8
53	60	10	30	1.00	0.27	0.69	51.0	13.8	35.2	51.0	39.3
54	60	20	20	1.00	0.50	0.50	50.0	25.0	25.0	50.0	50.0
55	60	30	10	1.00	0.69	0.27	51.0	35.2	13.8	51.0	60.7
56	60	40	0	1.00	0.80	0.00	55.6	44.4	0.0	55.6	72.2
57	70	0	30	1.00	0.00	0.60	62.5	0.0	37.5	62.5	31.3
58	70	10	20	1.00	0.24	0.45	59.2	14.2	26.6	59.2	43.8
59	70	20	10	1.00	0.45	0.24	59.2	26.6	14.2	59.2	56.2
60	70	30	0	1.00	0.60	0.00	62.5	37.5	0.0	62.5	68.8
61	80	0	20	1.00	0.00	0.40	71.4	0.0	28.6	71.4	35.7
62	80	10	10	1.00	0.22	0.22	69.4	15.3	15.3	69.4	50.0
63	80	20	0	1.00	0.40	0.00	71.4	28.6	0.0	71.4	64.3
64	90	0	10	1.00	0.00	0.20	83.3	0.0	16.7	83.3	41.7
65	90	10	0	1.00	0.20	0.00	83.3	16.7	0.0	83.3	58.3
66	100	0	0	1.00	0.00	0.00	100.0	0.0	0.0	100.0	50.0

Table 3 Inference results for fuzzy inputs of RGB colors

No.	Fuzzy input			Grade for fuzzy input			Inference output			Graphic output	
	r_i'	g_i'	b_i'	α_1'	α_2'	α_3'	r_o'	g_o'	b_o'	r_o'	uk'
1	0	0	100	0.19	0.19	1.00	13.6	13.6	72.7	13.6	20.5
2	0	10	90	0.20	0.35	1.00	12.8	22.6	64.6	12.8	29.0
3	0	20	80	0.21	0.51	1.00	12.2	29.8	58.0	12.2	35.9
4	0	30	70	0.22	0.68	1.00	11.8	35.5	52.7	11.8	41.4
5	0	40	60	0.24	0.84	1.00	11.5	40.3	48.1	11.5	46.1
6	0	50	50	0.26	1.00	1.00	11.4	44.3	44.3	11.4	50.0
7	0	60	40	0.24	1.00	0.84	11.5	48.1	40.3	11.5	53.9
8	0	70	30	0.22	1.00	0.68	11.8	52.7	35.5	11.8	58.6
9	0	80	20	0.21	1.00	0.51	12.2	58.0	29.8	12.2	64.1
10	0	90	10	0.20	1.00	0.35	12.8	64.6	22.6	12.8	71.0
11	0	100	0	0.19	1.00	0.19	13.6	72.7	13.6	13.6	79.5
12	10	0	90	0.35	0.20	1.00	22.6	12.8	64.6	22.6	24.1
13	10	10	80	0.37	0.37	1.00	21.3	21.3	57.5	21.3	31.9
14	10	20	70	0.39	0.54	1.00	20.3	28.0	51.7	20.3	38.2
15	10	30	60	0.42	0.71	1.00	19.6	33.5	46.9	19.6	43.3
16	10	40	50	0.45	0.89	1.00	19.2	38.0	42.9	19.2	47.5
17	10	50	40	0.45	1.00	0.89	19.2	42.9	38.0	19.2	52.5
18	10	60	30	0.42	1.00	0.71	19.6	46.9	33.5	19.6	56.7
19	10	70	20	0.39	1.00	0.54	20.3	51.7	28.0	20.3	61.8
20	10	80	10	0.37	1.00	0.37	21.3	57.5	21.3	21.3	68.1
21	10	90	0	0.35	1.00	0.20	22.6	64.6	12.8	22.6	75.9
22	20	0	80	0.51	0.21	1.00	29.8	12.2	58.0	29.8	27.1
23	20	10	70	0.54	0.39	1.00	28.0	20.3	51.7	28.0	34.3
24	20	20	60	0.57	0.57	1.00	26.7	26.7	46.5	26.7	40.1
25	20	30	50	0.61	0.76	1.00	25.8	32.0	42.2	25.8	44.9
26	20	40	40	0.70	1.00	1.00	26.0	37.0	37.0	26.0	50.0
27	20	50	30	0.61	1.00	0.76	25.8	42.2	32.0	25.8	55.1
28	20	60	20	0.57	1.00	0.57	26.7	46.5	26.7	26.7	59.9
29	20	70	10	0.54	1.00	0.39	28.0	51.7	20.3	28.0	65.7
30	20	80	0	0.51	1.00	0.21	29.8	58.0	12.2	29.8	72.9
31	30	0	70	0.68	0.22	1.00	35.5	11.8	52.7	35.5	29.6
32	30	10	60	0.71	0.42	1.00	33.5	19.6	46.9	33.5	36.3
33	30	20	50	0.76	0.61	1.00	32.0	25.8	42.2	32.0	41.8
34	30	30	40	0.81	0.81	1.00	30.9	30.9	38.3	30.9	46.3
35	30	40	30	0.81	1.00	0.81	30.9	38.3	30.9	30.9	53.7
36	30	50	20	0.76	1.00	0.61	32.0	42.2	25.8	32.0	58.2
37	30	60	10	0.71	1.00	0.42	33.5	46.9	19.6	33.5	63.7
38	30	70	0	0.68	1.00	0.22	35.5	52.7	11.8	35.5	70.4
39	40	0	60	0.84	0.24	1.00	40.3	11.5	48.1	40.3	31.7
40	40	10	50	0.89	0.45	1.00	38.0	19.2	42.9	38.0	38.1
41	40	20	40	1.00	0.70	1.00	37.0	26.0	37.0	37.0	44.5
42	40	30	30	1.00	0.81	0.81	38.3	30.9	30.9	38.3	50.0
43	40	40	20	1.00	1.00	0.70	37.0	37.0	26.0	37.0	55.5
44	40	50	10	0.89	1.00	0.45	38.0	42.9	19.2	38.0	61.9
45	40	60	0	0.84	1.00	0.24	40.3	48.1	11.5	40.3	68.3
46	50	0	50	1.00	0.26	1.00	44.3	11.4	44.3	44.3	33.5
47	50	10	40	1.00	0.48	0.89	42.3	20.3	37.4	42.3	41.4
48	50	20	30	1.00	0.61	0.76	42.2	25.8	32.0	42.2	46.9
49	50	30	20	1.00	0.72	0.61	42.9	30.8	26.3	42.9	52.3
50	50	40	10	1.00	0.89	0.45	42.9	38.0	19.2	42.9	59.4
51	50	50	0	1.00	1.00	0.26	44.3	44.3	11.4	44.3	66.5
52	60	0	40	1.00	0.24	0.84	48.1	11.5	40.3	48.1	35.6
53	60	10	30	1.00	0.45	0.71	46.3	20.7	33.0	46.3	43.8
54	60	20	20	1.00	0.57	0.57	46.5	26.7	26.7	46.5	50.0
55	60	30	10	1.00	0.71	0.42	46.9	33.5	19.6	46.9	56.9
56	60	40	0	1.00	0.84	0.24	48.1	40.3	11.5	48.1	64.4
57	70	0	30	1.00	0.22	0.68	52.7	11.8	35.5	52.7	38.1
58	70	10	20	1.00	0.42	0.54	51.0	21.3	27.6	51.0	46.8
59	70	20	10	1.00	0.54	0.39	51.7	28.0	20.3	51.7	53.9
60	70	30	0	1.00	0.68	0.22	52.7	35.5	11.8	52.7	61.9
61	80	0	20	1.00	0.21	0.51	58.0	12.2	29.8	58.0	41.2
62	80	10	10	1.00	0.39	0.37	56.7	22.3	21.0	56.7	50.6
63	80	20	0	1.00	0.51	0.21	58.0	29.8	12.2	58.0	58.8
64	90	0	10	1.00	0.20	0.35	64.6	12.8	22.6	64.6	45.1
65	90	10	0	1.00	0.37	0.20	63.8	23.6	12.6	63.8	55.5
66	100	0	0	1.00	0.19	0.19	72.7	13.6	13.6	72.7	50.0

fuzzy set A_k for fuzzy input I_n differed depending on whether or not I_n included the linear *edge* of A_k. The edges affected the nonlinear information processing. However, the edge effects do not consider in this study.

Figure 7 illustrates the relationship between the vertical value *uk* and the redness value r_o obtained from data (r_o', uk') in Tables 2. Filled circles indicate outputs for crisp inputs of colors corresponding to Fig. 3c, and open circles indicate crisp inputs of colors. The inference outputs (filled circles) for crisp inputs are not the same as the coordinates for the inputs (open circles) on the RGB color triangle. However six positions of *R*, *Y*, *G*, *C*, *B*, and *M* colors are no change. The other inference outputs (filled circles) for crisp inputs are grouped at the center of the RGB color triangle. This effect is causing from the shapes of membership function (like a triangular pyramid) and the computing of the center of gravity. The results are different from the results of previous studies [5]-[8]. Namely the gathering effects for crisp inputs using input fuzzy sets of triangular pyramid are not existed in previous studies [5]-[8].

Figure 8 also illustrates the relationship between the unknown value *uk* and the redness value r_o obtained from data (r_o', uk') in Table 3. Filled circles indicate outputs for fuzzy inputs of colors, corresponding to Fig. 3c. The inference outputs (filled circles) for fuzzy inputs are gathered at the center of the RGB color triangle. The circles are clearly different in this case. Vague color inputs to the RGB color triangle (Fig. 3a), the system outputs crisp color on the RGB color triangle (Fig. 3b), and also outputs crisp color on the graphical plane (Fig. 3c).

In Table 4, distances between crisp and fuzzy inference outputs are computed on the graphical plane. Distance 1 shows three distances among coordinates (r_o', uk') of *R*, *G*, and *B*. Distance 2 shows the distance between coordinates (r_o', uk') of crisp and fuzzy outputs for the six fundamental colors. Distance 3 shows the distances between the six fundamental colors and white for crisp inference outputs. The normalized distance (Ratio) is the ratio of Distance 2 to Distance 3. Ratios are divided into two groups. Group 1 is *R*, *G*, and *B* with a ratio of 0.41, and Group 2 is *Y*, *M*, and *C* with a ratio of 0.34. Inference outputs for fuzzy input at points *R*, *G*, and *B* have large normalized dis-

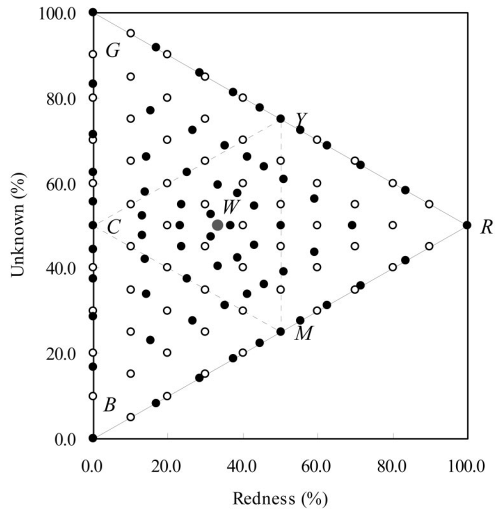

Fig.7 Inference outputs (*filled circles*) for crisp inputs (*open circles*) on the graphical plane. White (*large filled circle*) exists in the coordinates (33.3%, 50.0%)

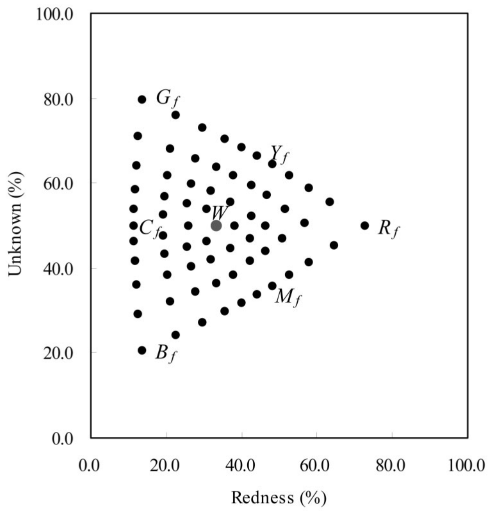

Fig.8 Inference outputs (*filled circles*) for fuzzy inputs on the graphical plane. White (*large filled circle*) exists in the coordinates (33.3%, 50.0%)

Table 4 Distances between crisp and fuzzy inference outputs on the graphical plane. Ratio = Distance 2 / Distance 3

Distance 1		No.	Distance 2		Distance 3		Ratio
B-G	100.0	1	B_f-B	24.6	*B-W*	60.1	0.409
		6	C_f-C	11.4	*C-W*	33.3	0.342
G-R	111.8	11	G_f-G	24.6	*G-W*	60.1	0.409
		46	M_f-M	10.3	*M-W*	30.0	0.342
		51	Y_f-Y	10.3	*Y-W*	30.0	0.342
R-B	111.8	66	R_f-R	27.3	*R-W*	66.7	0.409

tance (Ratio). But those at points Y, M, and C have small normalized distance (Ratio). It means that the gathering effects are strong, if Distance 3 from target color to white is large. Otherwise these are weak.

The chromaticity coordinates are denoted by r_o', g_o', b_o' and x, y, z. The transformation from R, G, and B to X_i, Y_i, and Z_i can be shown as follows [2]:

$$X_i = 2.77\, r_o' + 1.75\, g_o' + 1.13\, b_o' \quad (10)$$

$$Y_i = 1.00\, r_o' + 4.59\, g_o' + 0.06\, b_o' \quad (11)$$

$$Z_i = 0.00\, r_o' + 0.06\, g_o' + 5.59\, b_o' \quad (12)$$

The general definitions of the chromaticies x, y, z [1], [3] are:

$$x = X_i / (X_i + Y_i + Z_i) \quad (13)$$

$$y = Y_i / (X_i + Y_i + Z_i) \quad (14)$$

$$z = Z_i / (X_i + Y_i + Z_i) \quad (15)$$

where $x + y + z = 1$.

Tables 5 and 6 show the output of transformation from r_o, g_o, b_o to X_i, Y_i, Z_i and the chromaticies x and y, using Eqs. (10)-(14). In this case, chromaticity z (Eq. (15)) is not used.

Table 5 Transformed value and chromaticity for crisp inputs of six fundamental colors

No.	Transformed value			Chromaticity	
	X_i	Y_i	Z_i	x	y
1	288.2	15.3	1425.5	0.167	0.009
6	734.4	1185.8	1440.8	0.219	0.353
11	446.3	1170.5	15.3	0.273	0.717
46	994.5	270.3	1425.5	0.370	0.100
51	1152.6	1425.5	15.3	0.444	0.550
66	706.4	255.0	0.0	0.735	0.265

Table 6 Transformed value and chromaticity for fuzzy inputs of six fundamental colors

No.	Transformed value			Chromaticity	
	X_i	Y_i	Z_i	x	y
1	366.8	205.7	1038.5	0.228	0.128
6	405.9	554.4	638.3	0.254	0.347
11	460.2	887.9	205.7	0.296	0.571
46	491.4	253.2	633.2	0.357	0.184
51	543.5	633.2	169.3	0.404	0.470
66	613.8	347.2	196.6	0.530	0.300

Figure 9 illustrates the differences between crisp and fuzzy inputs on the chromaticity diagram. Only the six fundamental colors (filled circles) computed in this study show the movements in direction from coordinates (x, y) for crisp input to those for fuzzy input. The direction indicates toward white W (large filled circles), for example, R_f as the coordinate for fuzzy input lies midway between R and W as the coordinates for crisp inputs. The output (x, y) for fuzzy input is dislocated from the center of the conical fuzzy input (vague colors). Crisp input and its inference output for the six fundamental colors do not have the same coordinates. Each output for a fuzzy input is distant from the center of vague colors, although do not examine to compute for sixty colors, except for six fundamental colors. This implies that vague colors move toward the direction of white. It implies that the distance from inference outputs for crisp inputs to those for fuzzy inputs is also longer, if vagueness becomes larger. This means all the colors with or without vagueness becomes toward white direction, except for R, Y, G, C, B, and M. Because the average color value as the center of gravity is computed.

Table 7 shows the distances between crisp and fuzzy inference outputs on the chromaticity diagram. Distance 1, Distance 2, and Distance 3 are as described in Table 4. Distance 2 shows the distances between

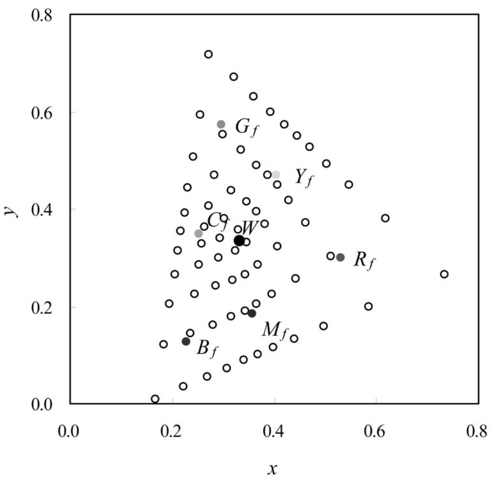

Fig.9 Inference outputs (*open circles*) for sixty-six crisp inputs and inference outputs (*filled circles*) for six conical fuzzy inputs on the chromaticity diagram. White (*large filled circle*) is located at the coordinates (0.33, 0.33)

Table 7 Distances between crisp and fuzzy inference outputs on the chromaticity diagram. Ratio = Distance 2 / Distance 3

Distance 1		No.	Distance 2		Distance 3		Ratio
B-G	0.734	1	B_f-*B*	0.134	*B-W*	0.365	0.366
		6	C_f-*C*	0.036	*C-W*	0.116	0.308
G-R	0.646	11	G_f-*G*	0.147	*G-W*	0.388	0.380
		46	M_f-*M*	0.084	*M-W*	0.236	0.358
		51	Y_f-*Y*	0.089	*Y-W*	0.243	0.366
R-B	0.623	66	R_f-*R*	0.207	*R-W*	0.407	0.510

the centers of six vague color inputs (*R*, *Y*, *G*, *C*, *B*, and *M*) and the fuzzy inference outputs. Distance 3 shows the distances among the six fundamental colors and white (crisp).

Table 4 shows the normalized distance (Ratio) between crisp and fuzzy inference outputs in graphical coordinates (*r*, *uk*) as follows:

$$(R_f\text{-}R) / (R\text{-}W) = (G_f\text{-}G) / (G\text{-}W) = (B_f\text{-}B) / (B\text{-}W) \quad (16)$$

$$(Y_f\text{-}Y) / (Y\text{-}W) = (C_f\text{-}C) / (C\text{-}W) = (M_f\text{-}M) / (M\text{-}W) \quad (17)$$

Table 7 shows the normalized distance (Ratio) between crisp and fuzzy inference outputs in chromaticity coordinates (*x*, *y*) as follows:

$$(R_f\text{-}R) / (R\text{-}W) > (G_f\text{-}G) / (G\text{-}W) > (B_f\text{-}B) / (B\text{-}W) \quad (18)$$

$$(Y_f\text{-}Y) / (Y\text{-}W) > (M_f\text{-}M) / (M\text{-}W) > (C_f\text{-}C) / (C\text{-}W) \quad (19)$$

In Eqs. (16) and (17), Group 1 as *R*, *G*, and *B* and Group 2 as *Y*, *M*, and *C* have different ratios (See Table 4). In Eqs. (18) and (19), six colors are ordered in each group (See Table 7).

4. Conclusions

The present paper proposes a fuzzy system that can extract crisp outputs of the RGB triangle (which is available for use in fuzzy set theory), a graphical system (which is easy to show via graphs), and chromaticity. It is difficult to construct such a fuzzy system on the chromaticity diagram directly, because the membership function of a triangular pyramid-like shape or cone is quite complicated on the nonlinear chromaticity diagram. The system also extracts, in a simple manner, the membership grades from the projection of a conical membership function of a vague color input. Three parameters associated with respective grades indicate vague colors and output the center of gravity as a crisp color value although the RGB triangle does not have a vertical attribute (on the unknown axis).

In the future, this system will help to ensure important color information (e.g. vagueness and color shading) in manufactured goods and art by reducing the confusion between colors that is often experienced by people.

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