Fuzzy Set Theoretical Approach to the RGB Color Triangle

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Abstract. The present study considers a fuzzy color system in which triangular pyramid-like membership functions are constructed on the RGB color triangle. This system can process a fuzzy input to an RGB system and output the center of gravity of three weights associated with respective grades. Triangular pyramid-like membership functions are applied to the RGB color triangle relationship. By treating three membership functions of redness, greenness, and blueness on the RGB color triangle, a target color can be easily obtained as the center of gravity of the output fuzzy set. In the present paper, the differences among fuzzy input, inference output, and chromaticity are described, and the relationship between inference outputs for crisp inputs and inference outputs for fuzzy inputs on the chromaticity diagram are shown for colors that have circular or elliptical vagueness areas.

1 Introduction

Additive color mixing occurs when two or three beams of differently colored light combine. It has been found that mixing just three additive primary colors, red, green and blue, can produce the majority of colors. In general, a color can be described by certain quantities, called the tristimulus values, r for the red component, g for the green component, and b for the blue component, as follows:

$$\text{color} = r + g + b \tag{1}$$

This is called the RGB color model. This concept allows colors to be represented by a planar diagram. The first step is to draw the red, green and blue components as the vertices of a color triangle, as in Fig. 1. The coordinates in the plane of the color triangle can specify various colors. The location given by the coordinates corresponds to the amounts of r, g and b that make up the color. The coordinates specifying the center of the color triangle represent the case in which the three primary colors are mixed in equal proportion and indicate the color white. Such representations are called chromaticity diagrams. The diagram represents hue and saturation but not lightness [9]. On the RGB color triangle, the percentages of redness, greenness, and blueness, where the total of the three attributes is equivalent to 100%, specify a color.

In the Natural Color System (NCS), a method similar to the fuzzy set theoretical method for obtaining hue expressions with vagueness has been reported by Sivik [4]. Using the fuzzy set theoretical method, a technique for acquiring tone expressions

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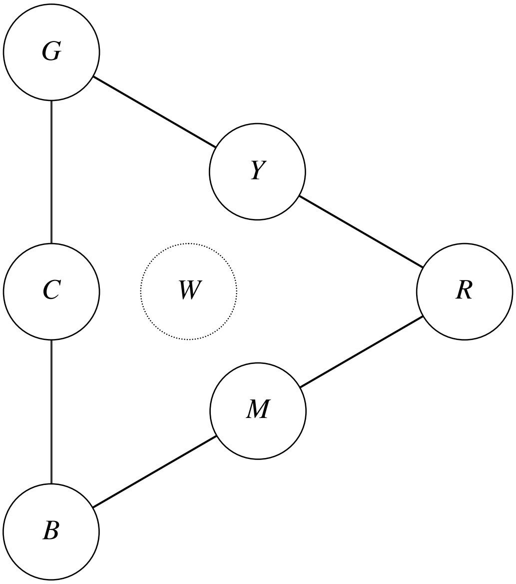

Fig. 1. A color triangle. A point in the plane of the triangular system represents the hue and saturation of a color.

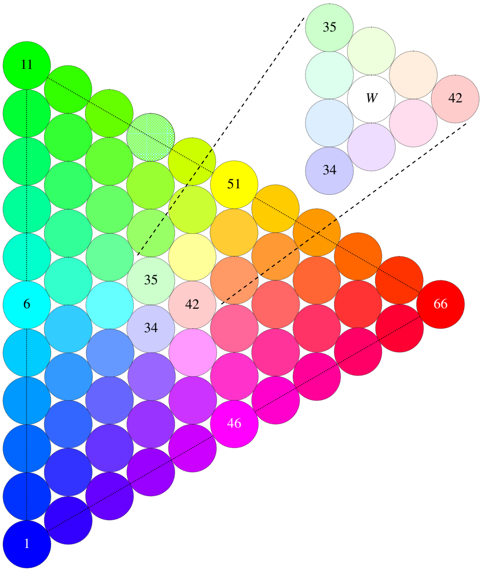

Fig. 2. Sixty-six crisp color inputs and white with six neighboring colors (*detail*) on the RGB color triangle

with vagueness on the NCS color triangle has been investigated by Sugano [5], [7]. In a recent study, the triangular membership functions of achromatic colors and conical membership functions of chromatic colors were used as vagueness [5], [7], which caused a gathering effect toward the center of the NCS tone triangle. In this previous study, fuzzy achromatic colors of triangular membership functions and fuzzy modified achromatic colors of conical membership functions were used on the NCS color triangle in a manner corresponding to the HLS (hue, lightness, and saturation) tone plane consisting of lightness and saturation. The vagueness effects of achromatic colors and modified achromatic colors (e.g., reddish, yellowish, greenish, and bluish achromatic colors) have been clarified [6], [8].

However, a technique for obtaining expressions of the RGB color triangle using the fuzzy set theoretical method has not been reported. In the present study, input fuzzy sets of a triangular pyramid-like shape with a plateau on the RGB triangle and fuzzy inputs of conical membership functions are examined. The RGB color triangle (plane) represents the hue and saturation of a color [9]. The six fundamental colors and white can be represented on the same color triangle (See Fig. 1). Vague colors on the RGB color triangle and chromaticity diagram are clarified. Such a system will help us to determine the average color value as the center of gravity of the attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

2 Methods and Results

The present study considers a system of the three primary colors, red, green, and blue (RGB), presented on an RGB color triangle. As Fig. 1 shows, blue, cyan, green, yellow, red, magenta, and white are abbreviated as B, C, G, Y, R, M, and W, respectively. Six fundamental color coordinates, e.g., (r_1, g_1, b_1), (r_6, g_6, b_6), (r_{11}, g_{11}, b_{11}), …, were selected, where r_n, g_n, and b_n are the red, green, and blue components, respectively, of the n^{th} color.

Figure 2 corresponds to the schematic diagram shown in Fig. 1. The color names in Fig. 2 are No.1: blue, No.6: cyan, No.11: green, No.46: magenta, No.51: yellow, and No.66: red. White is surrounded by six neighboring colors, as shown in the detail inset, and these seven colors are surrounded by No.34, No.35, and No.42.

Figure 3 illustrates input fuzzy sets of triangular pyramid-like shape, fuzzy input, and crisp output on the RGB color triangle, and crisp output on the graphical plane. The fuzzy rules are as follows (See Figs. 3 and 5):

$$R^1 : \text{IF } U \text{ is } A_1 \text{ THEN } V \text{ is } O_1 \tag{2}$$

$$R^2 : \text{IF } U \text{ is } A_2 \text{ THEN } V \text{ is } O_2 \tag{3}$$

$$R^3 : \text{IF } U \text{ is } A_3 \text{ THEN } V \text{ is } O_3 \tag{4}$$

where k is the rule number, A_k is a fuzzy set of inputs, O_k is a crisp set of outputs, $U = (r_i, g_i, b_i)$ are input parameters (variable), and $V = (r_o, g_o, b_o)$ are output parameters.

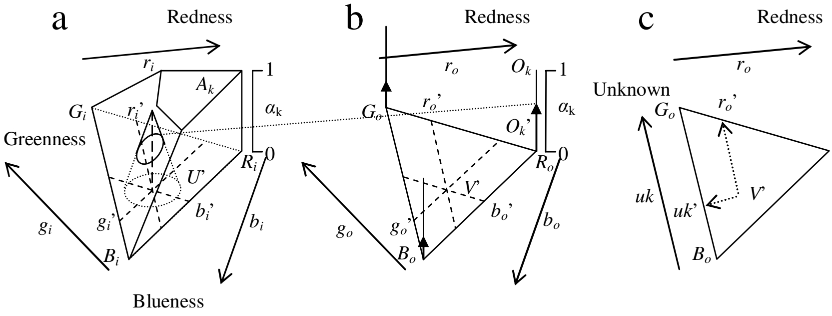

Fig. 3. Fuzzy system using the membership function of a triangular pyramid-like shape on the RGB color triangle

Here, U and V are fixed to these RGB parameters. A fuzzy set A_k of inputs shows a triangular pyramid-like shape at corner points R_i, G_i, and B_i, and a crisp set O_k of outputs of rule R^k is shown at corner points R_o, G_o, or B_o (a fuzzy set O_k' indicated by vertical arrows in Fig. 3b) on the color triangular, and the output is O_k if the input is A_k.

The fuzzy inference method is as follows. Let the inputs be $r_i = r_i$', $g_i = g_i$', and $b_i = b_i$'. 1) The input of rule R^k, grade $\alpha_k = A_k(U')$, where $k = 1, 2, 3$. 2) The output of rule R^k, and the α_k level-set is shown as a vertical filled allow. 3) $O_k' = \alpha_k O_k$, where O_k is fuzzy set in Fig. 3b. 4) The complete inference result O' of rules R^1, R^2, and R^3.

$$O' = \alpha_1 O_1 \cup \alpha_2 O_2 \cup \alpha_3 O_3 = O_1' \cup O_2' \cup O_3' \tag{5}$$

The output parameter, $V' = (r_o', g_o', b_o')$, corresponds to the coordinates of the central axis of the membership function of O', which is a de-fuzzification. In addition, in Fig. 3c, $V' = (r_o', uk')$ corresponds to a coordinate of the graphical system, where uk' (on the vertical axis) is calculated from g_o' and b_o'.

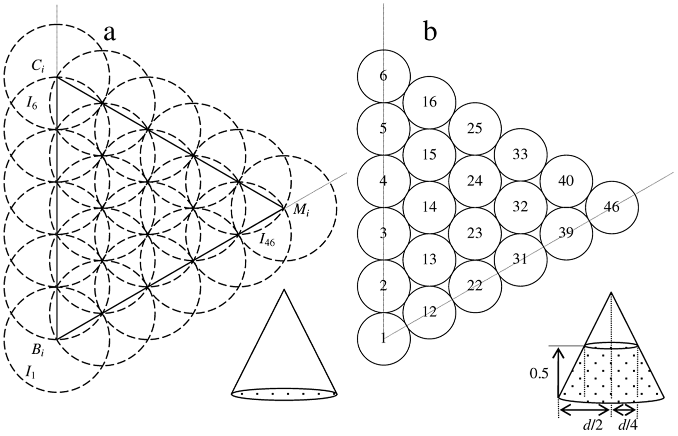

Fig. 4. Fuzzy inputs on part of the RGB color triangle and top areas of 0.5 level-sets indicated by number

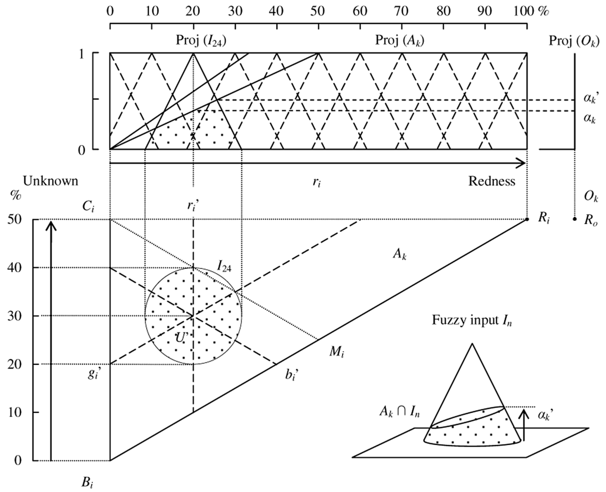

Fig. 5. Membership functions of triangular pyramid-like shape on the half of the RGB color triangle and one of sixty-six conical fuzzy inputs (*vague colors*)

Figure 4a (left) illustrates fuzzy inputs ($I_1 - I_{46}$) on the RGB color triangle as a triangle with color names (*B*, *C*, and, *M*). The fuzzy inputs are formed by conical membership functions, and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles. Figure 4b (right) shows the arrangement of numbers corresponding to the conical membership functions of Fig. 4a, and the numbers are shown inside circles representing the top of the 0.5 level set (bottom-right). The color names are No.1: blue, No.6: cyan, and No.46: magenta.

Figure 5 illustrates half of the RGB color triangle as a base of input fuzzy set A_k and one of the sixty-six conical fuzzy inputs ($I_1 - I_{66}$) on the RGB color triangle. The triangular membership function Proj(I_{24}) on the redness axis is one of eleven projections of the sixty-six fuzzy inputs ($I_1 - I_{66}$) by the rays from the lower part, and the triangular membership function Proj(I_n) on the unknown axis is not used in the present study.

The intersection of input fuzzy set A_k for fuzzy input I_n is $A_k \cap I_n$. (See the dotted area at the bottom-right of Fig. 5.) Grade α_k' = height ($A_k \cap I_n$). If the input is crisp, α_k' becomes α_k. R_o is the new red. Proj(O_k) is a projection of an output crisp set at the corner point R_o (See Fig. 3b).

What happens if a vague color is input into the RGB system? The system considered in the present study can translate input data *U* of a vague color to output data *V* of a simple color on the RGB color triangle. The fuzzy input (No.24) on the RGB is made up of the center $U' = (r_i', g_i', b_i') = (20, 20, 60)$ in % and the diameter d = 23.0% of the basal plane (circle) of the cone indicated vagueness.

Figure 6 illustrates the relationship between the unknown value *uk* and the redness value r_o obtained from data (r_o', *uk*'). Filled circles indicate outputs for crisp inputs of colors corresponding to Fig. 3c, and open circles indicate crisp inputs of colors. The inference outputs (filled circles) for crisp inputs are not the same as the coordinates

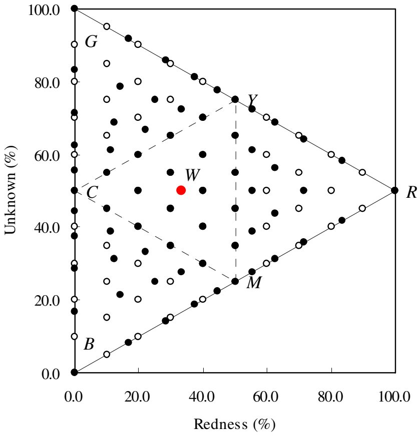

Fig. 6. Inference outputs (*filled circles*) for crisp inputs (*open circles*) on the graphical plane. White (*filled circle*) exists in the coordinates (33.3%, 50.0%).

for the inputs (open circles) on the RGB color triangle. However the positions of twenty-one colors (circles) are no change on the inside of YMC triangle and three positions of RGB colors are also no change. The inference outputs (filled circles) for crisp inputs are grouped at the center of the RGB color triangle.

Figure 7 also illustrates the relationship between the unknown value *uk* and the redness value r_o. Filled circles indicate outputs for fuzzy inputs of colors, corresponding to Fig. 3c. The inference outputs (filled circles) for fuzzy inputs are gathered at the center of the RGB color triangle. The circles are clearly different in this case. Vague color inputs to the RGB color triangle (Fig. 3a), the system outputs crisp color on the RGB color triangle (Fig. 3b), and also outputs crisp color on the graphical plane (Fig. 3c).

The chromaticity coordinates are denoted by r_o', g_o', b_o' and x, y, z. The transformation from R, G, and B to X_i, Y_i, and Z_i can be shown as follows [2]:

$$X_i = 2.77\, r_o' + 1.75\, g_o' + 1.13\, b_o' \tag{6}$$

$$Y_i = 1.00\, r_o' + 4.59\, g_o' + 0.06\, b_o' \tag{7}$$

$$Z_i = 0.00\, r_o' + 0.06\, g_o' + 5.59\, b_o' \tag{8}$$

The general definitions of the chromaticies x, y, z [1], [3] are:

$$x = X_i / (X_i + Y_i + Z_i) \tag{9}$$

$$y = Y_i / (X_i + Y_i + Z_i) \tag{10}$$

$$z = Z_i / (X_i + Y_i + Z_i) \tag{11}$$

where $x + y + z = 1$.

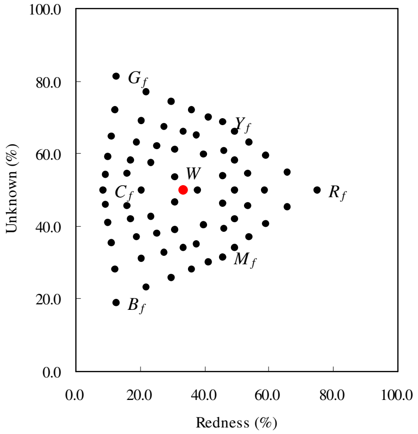

Fig. 7. Inference outputs (*filled circles*) for fuzzy inputs on the graphical plane. White (*filled circle*) exists in the coordinates (33.3%, 50.0%).

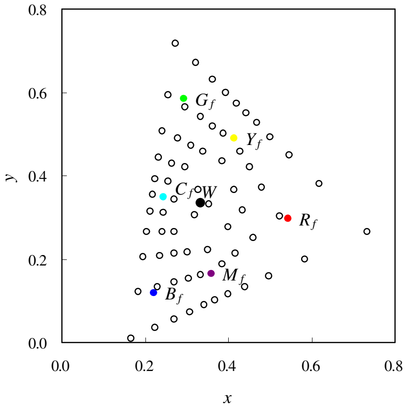

Fig. 8. Transformed inference outputs (*open circles*) for sixty-six crisp inputs and transformed inference outputs (*filled circles*) for six conical fuzzy inputs on the chromaticity diagram. White (*filled circle*) is located at the coordinate (0.33, 0.33).

Figure 8 illustrates the differences between crisp and fuzzy inputs. Only the six fundamental colors (filled circles) show changes in direction from coordinates (x, y) for crisp input to those for fuzzy input. The direction indicates toward white W (central filled circles), for example, R_f lies midway between R and W. The output (x, y) for fuzzy input is dislocated from the center of the conical fuzzy input (vague colors). Crisp input and its inference output for the six fundamental colors do not have the same coordinates. Each output for a fuzzy input is distant from the center of vague colors, although do not examine for sixty colors, except for six fundamental colors.

This implies that vague colors move toward the direction of white, as such vague colors are input to the fuzzy system.

3 Conclusions

The present paper proposes a fuzzy system that can extract crisp outputs of the RGB triangle (which is available for use in fuzzy set theory), a graphical system (which is easy to show via graphs), and chromaticity. It is difficult to construct such a fuzzy system on the chromaticity diagram directly, because the membership function of a triangular pyramid-like shape or cone is quite complicated on the nonlinear chromaticity diagram. The system also extracts, in a simple manner, the membership grades from the projection of a conical membership function of a vague color input. Three parameters associated with respective grades indicate vague colors and output the center of gravity as a crisp color value although the RGB triangle does not have a vertical attribute (on the unknown axis).

In the future, this system will help to ensure important color information (e.g. vagueness and color shading) in manufactured goods and art by reducing the confusion between colors that is often experienced by people.

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