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Fuzzy Set Theoretical Approach to the RGB Tone Triangular System

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***Abstract*—The present study considers a fuzzy color system in which two membership functions are constructed on the tone triangle. This system can process a fuzzy input to a tone triangular system and output to an RGB triangular system. Two membership functions (*anti-blackness* and *whiteness*) are applied to the tone triangle relationship. By treating the input parameters of chromaticness, whiteness, and blackness on the tone triangle, a target color can be easily obtained as the center of gravity of the output fuzzy set. In the present paper, the differences between fuzzy input and inference output are described, and the relationship between inference outputs for crisp inputs and inference outputs for fuzzy inputs on the RGB triangular system are shown by the characteristics between chromaticness, whiteness, and blackness as the inputs and redness as one of outputs.**

I. Introduction

In the Natural Color System (NCS), a method similar to the fuzzy set theoretical method for obtaining hue expressions with vagueness has been reported by Sivik [1]. Using the fuzzy set theoretical method, a technique for acquiring tone expressions with vagueness on the NCS color triangle has been investigated by Sugano [2]. The triangular membership functions of achromatic colors and conical membership functions of chromatic colors were used as vagueness, which caused a gathering effect toward the center of the NCS tone triangle [2]. In addition, the fuzzy achromatic colors of triangular membership functions and fuzzy modified achromatic colors of conical membership functions were used on the NCS color triangle in a manner corresponding to the HLS (hue, lightness, and saturation) tone plane consisting of lightness and saturation. The vagueness effects of achromatic colors and modified achromatic colors (e.g., reddish, yellowish, greenish, and bluish achromatic colors) have been clarified [3].

In the recent studies [4], [5], the relationship between input fuzzy sets with a plateau on the RGB triangle and fuzzy inputs of conical membership functions was examined. The RGB color triangle (plane) represents the hue and saturation of a color. The six fundamental colors and white can be represented on the same color triangle (See Fig. 1b). Vague colors on the RGB color triangle and chromaticity diagram were clarified.

However, a technique for obtaining expressions of the tone triangle in the RGB system using the fuzzy set theoretical method has not been reported. In the present study, the relationship between two input fuzzy sets on the tone triangle (See Fig. 1a) and fuzzy inputs of conical membership functions is examined. The six fundamental colors (and white) can be represented on the RGB color triangle. Vague colors on the RGB color triangle are clarified. Such a system will help us to determine the average color value as the center of gravity of the attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

II. Methods

The present study considers a system of the three primary colors, red, green, and blue (RGB), presented on an RGB color triangle. As Fig. 1b shows, blue, cyan, green, yellow, red, magenta, and white are abbreviated as *B*, *C*, *G*, *Y*, *R*, *M*, and *W*, respectively. Six fundamental color coordinates, e.g., (r_1, g_1, b_1), (r_6, g_6, b_6), (r_{11}, g_{11}, b_{11}), …, were selected, where r_n, g_n, and b_n are the red, green, and blue components, respectively, of the n^{th} color. When the hue of a color is fixed on the RGB color triangle in Fig. 1b (e.g., red, green, and blue), the color exists on the tone triangle in Fig. 1a. As Fig. 1a shows, maximum chroma, white, and schwarz (black) are abbreviated as *C*, *W*, *S*, respectively.

Figure 2 corresponds to the schematic diagram shown in Fig. 1. The color names or modifiers in Fig. 2b are No.1: blue, No.6: cyan, No.11: green, No.51: yellow, No.66: red, No.46: magenta. In detail No.104 is white. Those in Fig. 2a are No.1: *S*, schwarz (black), No.6: gray, No.11: *W*, white, No.46: dark (or deep), No.51: light (or pale), No.66: *C*, maximum chroma (e.g., vivid red) [1], and dull is not fixed.

Figure 3 illustrates input fuzzy sets, fuzzy input on the tone triangle, crisp output, fuzzy output on the RGB color triangle, and crisp output on the graphical plane.

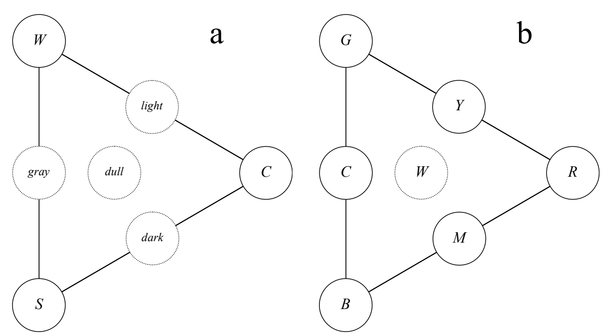

Fig. 1. A tone triangle (*a*). A point in the plane of the triangular system represents the lightness and saturation of a color. *S* is black. *C* is maximum chroma of each hue. A color triangle (*b*). A point in the plane of the triangular system represents the hue and saturation of a color. *C* is cyan.

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The fuzzy rules are as follows (See Figs. 3 and 6):

R^k : IF U is A_j THEN V is O_k (1)

, where k is the rule number (k = 1, 2, 3) corresponding to the components of r, g and b, A_j is a fuzzy set of inputs (j = 1, 2), O_k is a crisp set of outputs, k is not corresponding to j. U = (c, w, s) are input parameters (variable), and V = (r, g, b) are output parameters. Here, U is fixed to tone parameters and V is fixed to RGB parameters. A fuzzy set A_j of inputs shows a triangular pyramid shape at corner points C, W, and S, and a crisp set O_k of outputs of rule R^k is shown at corner points R, G, or B (a fuzzy set O_k' indicated by vertical arrows in Fig. 3b) on the RGB color triangle, and the output is O_k if the input is A_j. Relationship between A_j and O_k is shown in Table I. Table I shows the fuzzy rules of six fundamental colors. Three rules are composed of two membership functions in this case. See Figs. 3 and 4.

TABLE I
FUZZY RULES FOR SIX FUNDAMENTAL COLORS

Hue color	Input fuzzy set			Output crisp set		
	R^1	R^2	R^3	R^1	R^2	R^3
Red	A_1	A_2	A_2	O_1	O_2	O_3
Green	A_2	A_1	A_2	O_1	O_2	O_3
Blue	A_2	A_2	A_1	O_1	O_2	O_3
Yellow	A_1	A_1	A_2	O_1	O_2	O_3
Cyan	A_2	A_1	A_1	O_1	O_2	O_3
Magenta	A_1	A_2	A_1	O_1	O_2	O_3

The fuzzy inference method is as follows. Let the inputs be $c = c'$, $w = w'$, and $s = s'$. $U' = (c', w', s')$

1) The input of rule R^k, grade $\alpha_k = A_j(U')$, where k and j are shown in Table I.

2) The output of rule R^k, the α_k level-set is shown as a vertical allow.

3) $O_k' = \alpha_k O_k$, where O_k' is fuzzy sets and O_k is crisp sets in Fig. 3b. The complete inference results O' of rules R^1, R^2, and R^3.

$$O' = \alpha_1 O_1 \cup \alpha_2 O_2 \cup \alpha_3 O_3 = O_1' \cup O_2' \cup O_3' \quad (2)$$

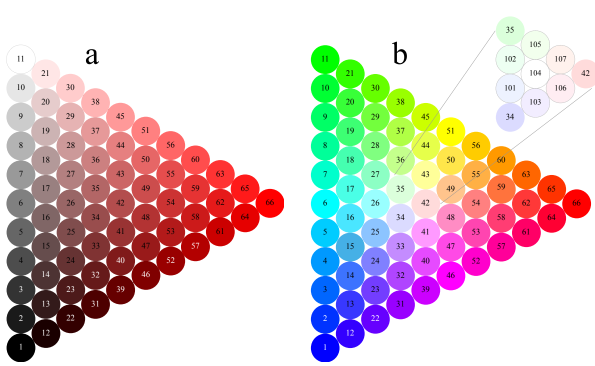

Fig. 2. Sixty-six crisp color inputs on the tone triangle (*a*). Sixty-six crisp color inputs and white with six neighboring colors (*detail*) on the RGB color triangle (*b*).

The output parameter, V' = (r', g', b'), corresponds to the coordinates of the central axis of the membership function of O'. In addition, in Fig. 3c, V' = (r', uk_o') corresponds to a coordinates of the graphical system, where uk_o' (on the vertical axis) is calculated from g' and b'. uk_o' shows a value (as distance from B) on the line B-G.

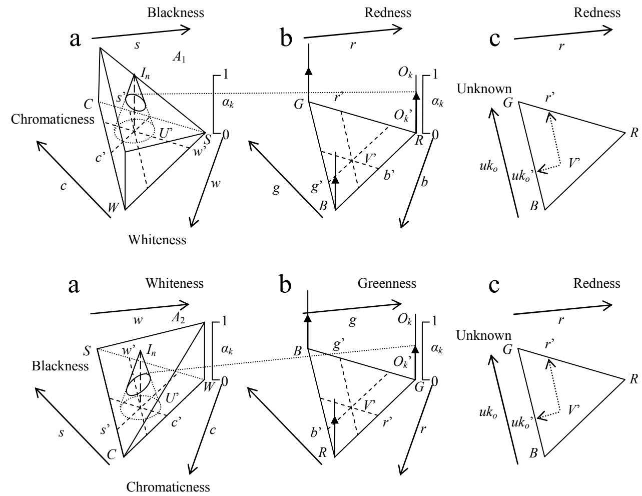

Fig. 3. Fuzzy system using the membership function of input fuzzy sets A_j, conical fuzzy input I_n on the tone triangle and output crisp sets O_k on the RGB color triangle. *a* (upper trace): input fuzzy set A_1 (*anti-blackness*) and *a* (lower trace): input fuzzy set A_2 (*whiteness*).

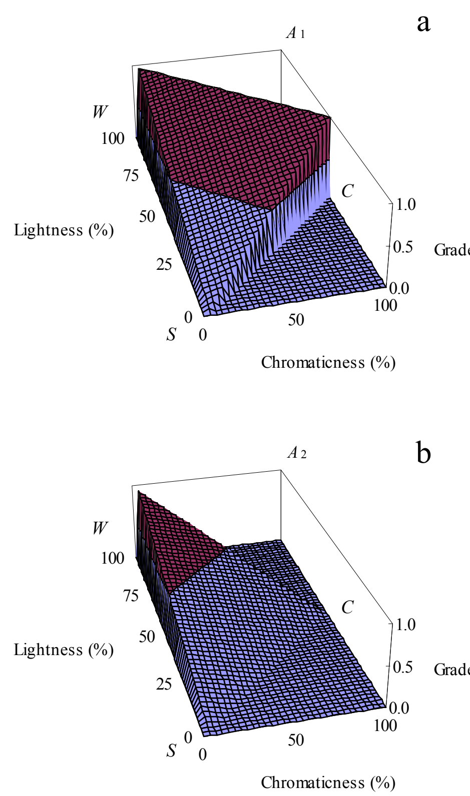

Fig. 4. Input fuzzy sets A_j and the membership functions μ_j. *a*: μ_1 (c, uk_3) of A_1 (*anti-blackness*) and *b*: μ_2 (c, uk_3) of A_2 (*whiteness*) on the tone triangle. uk_3 is equal to the lightness.

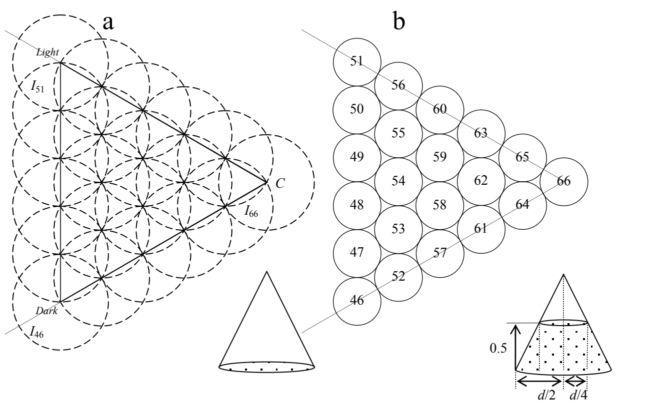

Fig. 5. Fuzzy inputs on part of the tone triangle (*a*) and top areas of 0.5 level-sets indicated by number (*b*). The diameter (d = 23.0%) of the basal plane (*circle*) of the cone indicated vagueness.

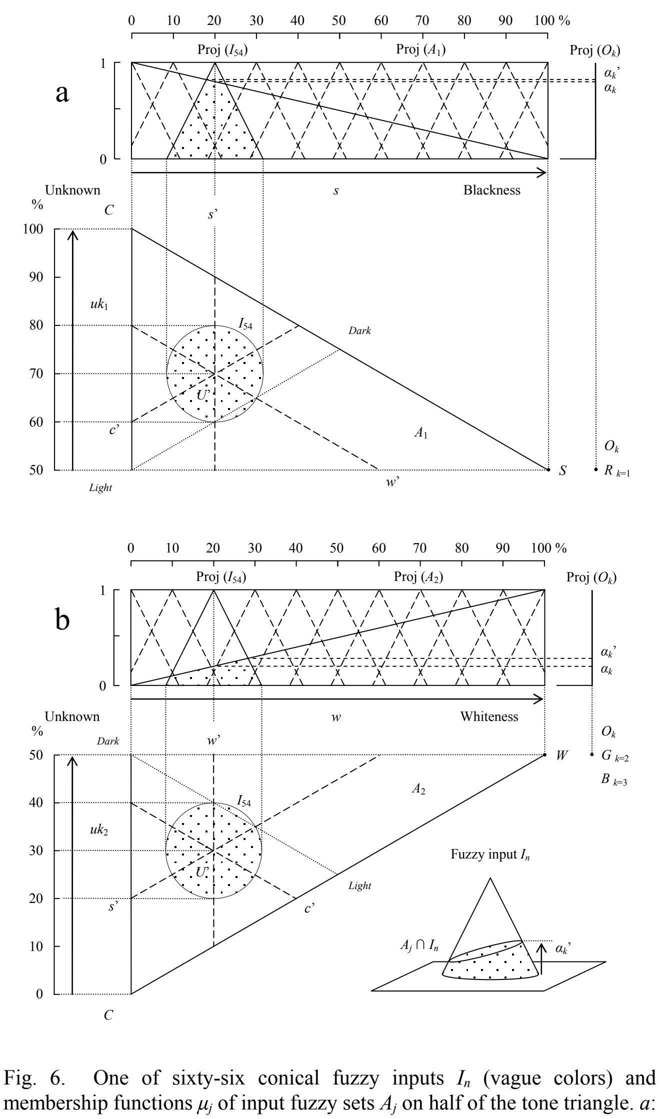

Fig. 6. One of sixty-six conical fuzzy inputs I_n (vague colors) and membership functions μ_j of input fuzzy sets A_j on half of the tone triangle. *a*: μ_1 (s, uk_1) = − 0.01s + 1 on the projection of A_1. *b*: μ_2 (w, uk_2) = 0.01w on the projection of A_2.

TABLE II
MEMBERSHIP VALUE OF INPUT FUZZY SET A_j ON THE TONE TRIANGLE

Color	Color coordinate			Membership value μ_j			
	c	w	s	j=1	j=2	j=3	j=4
C	1.00	0.00	0.00	1.00	0.00	0.50	0.25
W	0.00	1.00	0.00	1.00	1.00	1.00	1.00
S	0.00	0.00	1.00	0.00	0.00	0.00	0.00

C is maximum chroma of each hue. S is black.

Table II shows the membership value μ_j (c', w', s') of input fuzzy set A_j on the tone triangle. In Fig. 4 μ_j (c', w', s') is equal to μ_j (c', uk_3'). uk_3 is equal to the lightness. The membership function μ_j was based on the three additive primary colors in this study. μ_j is corresponding to A_j in Figs. 4 and 10. See Appendix.

Figure 4 shows the membership function of input fuzzy sets A_1 (*anti-blackness*) in *a* and A_2 (*whiteness*) in *b*. See also Table I, Fig. 3, and Fig. 6.

Figure 5a illustrates twenty-one fuzzy inputs (I_{46} – I_{66}) on part of the tone color triangle with color names: dark (or deep), light (or pale), and C. The fuzzy inputs are formed by conical membership functions, and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles. Fig. 5b shows the arrangement of numbers corresponding to the conical membership functions of Fig. 5a, and the numbers are shown inside circles representing the top of the 0.5 level-set (bottom-right). The color names or modifiers are No.46: dark (or deep), No.51: light (or pale) and No.66: C.

Figure 6 illustrates half of the tone triangle as a base of input fuzzy set A_j and one of the sixty-six conical fuzzy inputs (I_1 – I_{66}) on the tone triangle. In the input fuzzy set A_1 (as *anti-blackness* in *a*), slope line shows a projection of line between S with membership value $\mu_1 = 0$ and W with $\mu_1 = 1$ (or between S with membership value $\mu_1 = 0$ and C with $\mu_1 = 1$ on the other side) and in the input fuzzy set A_2 (as *whiteness* in *b*) slope line shows a projection of line between W with value $\mu_2 = 1$ and C with value $\mu_2 = 0$ (or between W with value $\mu_2 = 1$ and S with $\mu_2 = 0$ on the other side). See also Table 1 and Fig. 4. The triangular membership function Proj (I_{54}) on the blackness axis (*a*) and Proj (I_{54}) on the whiteness axis (*b*) is one of eleven projections of the sixty-six fuzzy inputs (I_1 – I_{66}) by the rays from the lower part, and the triangular membership function Proj (I_{54}) on the unknown axis (uk_1 and uk_2) is not used in this study.

An input fuzzy set A_1 of *anti-blackness* can be characterized by the following membership function:

$$\mu_1 (s, uk_1) = -0.01s + 1 \quad (3)$$

where 0.01 is slope of projection. See Fig. 6a.

The limitations of uk_1 (on C-W side) are as follows:

$$50 \geq uk_1 \geq s/2 \quad (4)$$

$$50 < uk_1 \leq -(s/2) + 100 \quad (5)$$

An input fuzzy set A_2 of *whiteness* can be characterized by the following membership function:

$$\mu_2 (w, uk_2) = 0.01w \quad (6)$$

where 0.01 is slope of projection. See Fig. 6b. The limitations of uk_2 (on *S*-*C* side) are as follows:

$50 \geq uk_2 \geq w/2$ (7)

$50 < uk_2 \leq -(w/2) + 100$ (8)

An input fuzzy set A_3 of *lightness* can be characterized by the following membership function:

$\mu_3(c, uk_3) = 0.01uk_3$ (9)

where 0.01 is slope of projection on uk_3. The limitations of uk_3 (on *W*-*S* side) are as follows:

$50 \geq uk_3 \geq c/2$ (10)

$50 < uk_3 \leq -(c/2) + 100$ (11)

An input fuzzy set A_4 of 15 *degrees lightness* can be characterized by the following membership function:

$\mu_4(uk_5, uk_4) = 0.01uk_4$ (12)

where uk_5 and uk_4 rotate 15 degrees for c and uk_3 at the corner *S*. 0.01 is slope of projection on uk_4. The limitations of uk_3 (on *W*-*S* side) are as follows:

$50 \geq uk_3 \geq c/2$ (13)

$50 < uk_3 \leq -(c/2) + 100$ (14)

$c \geq 0$ (15)

The details of A_3 and A_4 are not shown in Fig. 6

III. Results and Discussion

What happens if a vague color is input into the tone triangular system? The system considered in the present study can translate input data *U* of a vague color to output data *V* of a simple color on the RGB color triangle.

The intersection of input fuzzy set A_j for fuzzy input I_n is $A_j \cap I_n$. (See the dotted area at the bottom-right of Fig. 6b.) Grade α_k' = height ($A_j \cap I_n$). If the input is crisp, α_k' becomes α_k. *R* in Fig. 6a is the red, *G* or *B* in Fig. 6b is the green or the blue as output. Proj (O_k) is a projection of an output crisp set at the corner point *R*, *G*, or *B* (See Fig. 3b).

The fuzzy input (No.54) on the tone triangle is made up of the center *U*' = (*c*', *w*', *s*') = (60, 20, 20) in % and the diameter *d* = 23.0% of the basal plane (circle) of the cone indicated vagueness.

In previous study [2] the intersection of input fuzzy set A_j for fuzzy input I_n differed depending on whether or not I_n included the linear *edge* of A_j. The edges affected the nonlinear information processing. However, the edge effects do not consider in this study.

Figure 7a illustrates the relationship between the vertical value uk_o and the redness value *r* obtained from data (*r*', uk_o'). The circles indicate outputs for crisp inputs of colors in Fig. 7a, corresponding to Fig. 3c. The inference outputs for crisp inputs are grouped at the center of the RGB color triangle. This effect is causing from shapes of membership function (triangular pyramid) and computing of the center of gravity. The results are the same as the results of previous study [2]. Namely the gathering effects for crisp inputs using input fuzzy sets of triangular pyramid are existed in previous study [2].

Figure 7b also illustrates the relationship between the unknown value uk_o and the redness value *r* obtained from data (*r*', uk_o'). The circles indicate outputs for fuzzy inputs of colors, corresponding to Fig. 3c. The inference outputs for fuzzy inputs are also gathered at the center of the RGB color triangle. The inference outputs for crisp inputs in a are not the same as the coordinates for fuzzy inputs in b. Vague color inputs to the tone triangle (Fig. 3a), the system outputs crisp color on the RGB color triangle (Fig. 3b), and also outputs crisp color on the graphical plane (Fig. 3c).

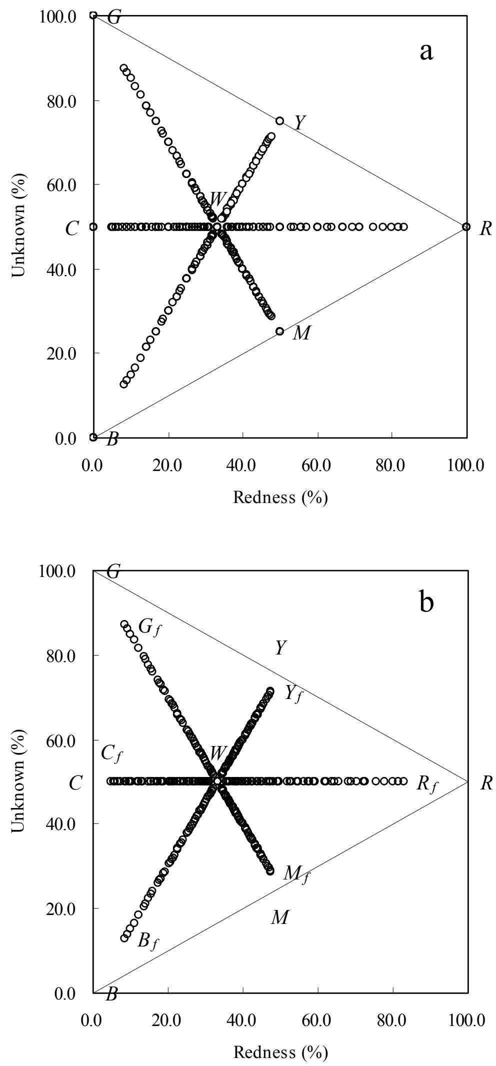

Fig. 7. Inference outputs for crisp inputs (*a*) and inference outputs for fuzzy inputs (*b*) on the graphical plane. Suffix *f* shows fuzzy inference output. White exists in the coordinates (33.3%, 50.0%).

Figures 8 and 9 illustrate the relationship between the chromaticness, the whiteness, the blackness as the inputs and the redness as one of the outputs. The outputs for crisp (Fig. 8) and fuzzy inputs (Fig. 9) are shown herein.

a shows the relationship between chromaticness and redness. With the increasing chromaticness the redness increases 100% for crisp (not like a leaf shape in Fig. 8) and 82.9% for fuzzy inputs (like a leaf shape in Fig. 9). With the decreasing chromaticness that redness (as the output) converges 33.3% (*W*-*S*) as the achromatic colors.

b shows the relationship between whiteness and redness.

c shows the relationship between blackness and redness.

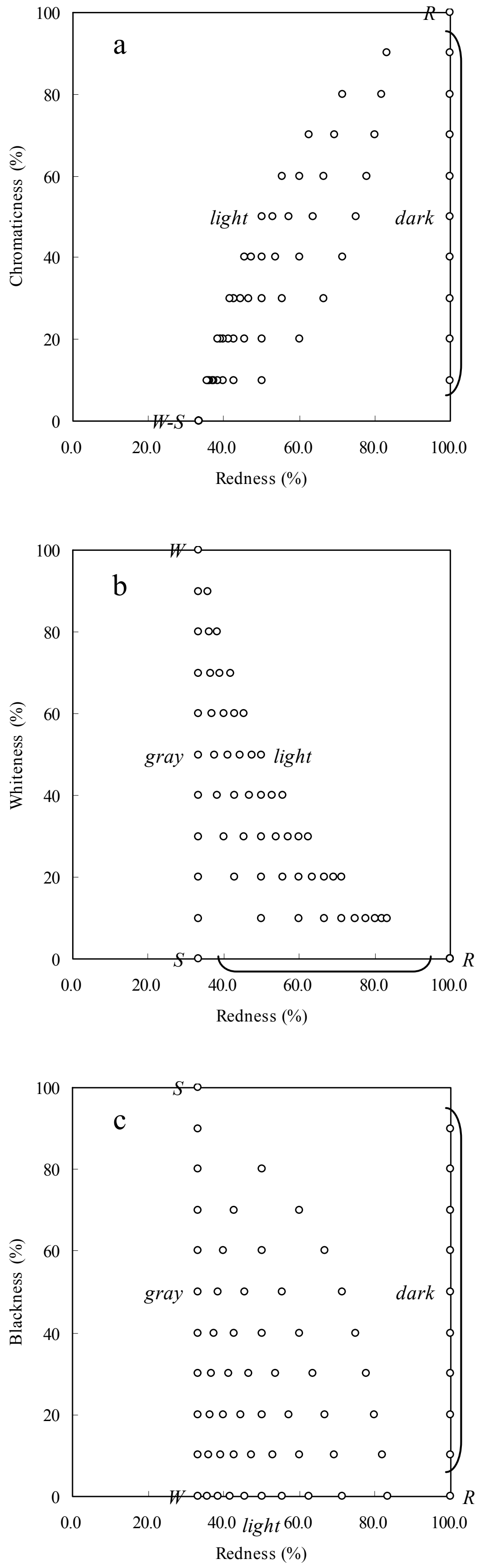

Fig. 8. Relationship between chromaticness (*a*), whiteness (*b*), and black -ness (*c*) as crisp inputs and redness as the inference outputs. *W-S* shows achromatic colors (No.1 - 11).

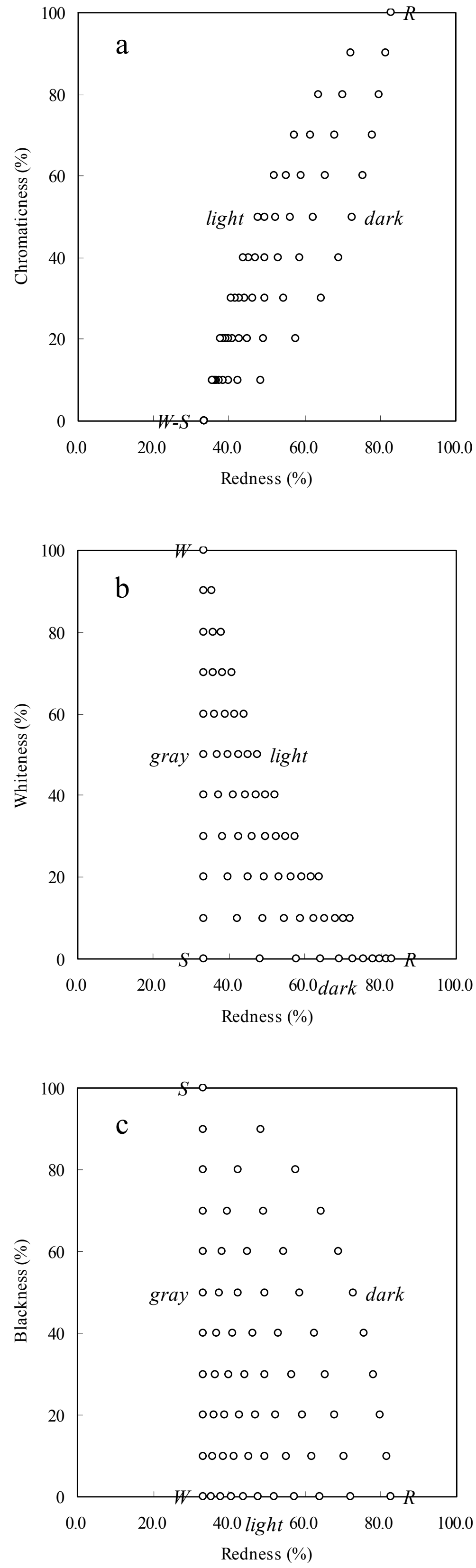

Fig. 9. Relationship between chromaticness (*a*), whiteness (*b*), and black -ness (*c*) as fuzzy inputs and redness as the inference outputs. *W-S* shows achromatic colors (No.1 - 11).

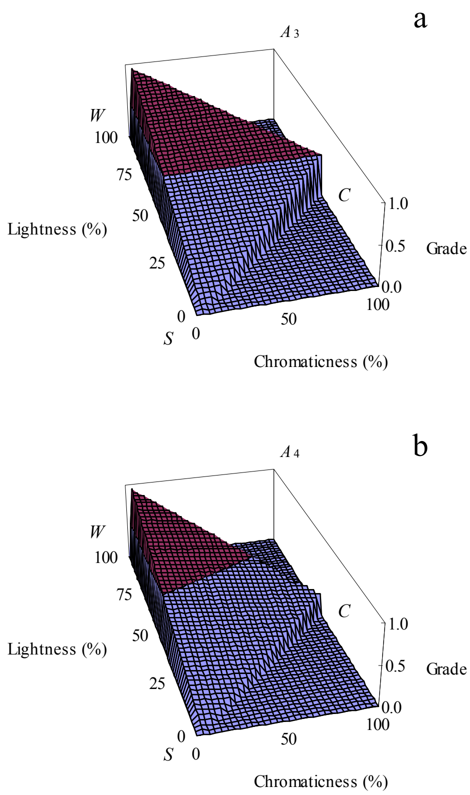

Fig. 10. Input fuzzy sets A_j and the membership functions μ_j. *a*: μ_3 (*c*, uk_3) of A_3 (*lightness*) and *b*: μ_4 (*c*, uk_3) of A_4 (15 *degrees lightness*) on the tone triangle. uk_3 is equal to the lightness

TABLE III
FUZZY RULES FOR THREE TYPICAL COLORS

Hue color	Input fuzzy set			Output crisp set		
	R^1	R^2	R^3	R^1	R^2	R^3
Orange	A_1	A_3	A_2	O_1	O_2	O_3
Lime	A_3	A_1	A_2	O_1	O_2	O_3
Brown	A_1	A_4	A_4	O_1	O_2	O_3

A_3 and A_4 are shapes between A_1 and A_2. Membership values of A_3 and A_4 are 0.5 and 0.25 in the coordinate (1, 0, 0). See Table II.

With the increasing whiteness (or blackness) the redness converges on the white (or black) as the redness of 33.3%. The envelope curves (*R-dark-W-S*, *R-light-W-S* in *a*, *W-light-R* in *b*, *S-dark-R* in *c*) are similar (Fig. 9). Whiteness (*b*) and blackness (*c*) are different characteristics, because the characteristics are caused from the shape of membership functions.

In the crisp inputs (Fig. 8) when w = 0 nine colors with parenthesis behave *rare* outputs. Those (in *a*) show outputs of 100%. Those (in *b*) show outputs of 100%. Nine colors with parenthesis converge to "*R*" coordinate. The output "*R*" displays including to nine colors (invisible). Those (in *c*) show the outputs of 100% excluding to "*R*" coordinate. The outputs of the redness for fuzzy inputs show natural shape (Fig. 9). However the outputs for crisp inputs, especially only nine colors show unnatural shapes (Fig. 8). This is because of the membership values μ_2 = 0 from *S* to *C* (See A_2 in Fig. 4).

Figure 10 shows the membership function of input fuzzy sets A_3 (*lightness*) and A_4 (15 *degrees lightness*). See also Table III.

Table III shows the fuzzy rules of three typical colors. Three rules are composed of two or three membership functions in this case. Orange is yellow red. Lime is yellow green.

IV. CONCLUSIONS

The present paper proposes a fuzzy system that can extract crisp outputs of the RGB triangle (which is available for use in fuzzy set theory) and a graphical system (which is easy to show via graphs). The system also extracts, in a simple manner, the membership grades from the projection of a conical membership function of a vague color input. Three parameters associated with respective grades indicate vague colors and output the center of gravity as a crisp color value although the RGB triangle does not have a vertical attribute (on the unknown axis). The relationship between the chromaticness, the whiteness, the blackness as the inputs and the redness as the output is clarified.

In the future, this system will help to ensure important color information (e.g. vagueness and color shading) in manufactured goods and art by reducing the confusion between colors that is often experienced by people.

In addition, the membership values (on the tone triangle [7] or on the color triangle [6]) as human subjectivity are analyzed.

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