Fuzzy Natural Color System Using Membership Function of Triangular Pyramid on Color Triangle

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Abstract: *The present study considered a fuzzy natural color system (NCS) in which triangular pyramid membership functions are constructed on the NCS color triangle. This system can process a fuzzy input to an NCS and output to a center of gravity of three weights associated with respective grades. Triangular membership functions are applied to the hue, and triangular pyramid membership functions are applied to the NCS color triangle relationship. By treating three membership functions of blackness, whiteness, and chromaticness on the NCS color triangle, a target color as a center of gravity of the output fuzzy set can be easily obtained. I will show herein the difference between crisp inputs and fuzzy inputs.*

Keywords *fuzzy set theoretical approach, natural color system (NCS), vague color, NCS color triangle, triangular pyramid membership function, conical membership function, and HLS (hue, lightness, saturation) system*

1. Introduction

The NCS (Natural Color System) color descriptive system and atlas were developed in Sweden [3]. In the NCS, a method for obtaining hue expressions has been reported by Sivik (1997). However, a technique for obtaining expressions of the NCS color triangle using the fuzzy set theoretical method has not been studied.

In this study, fuzzy inputs of conical membership functions are used on the NCS color triangle corresponding to the HLS tone plane consisted of lightness and saturation [5]-[8]. A vague color is clarified. Such a system will help us to determine the average color value as a center of gravity of attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

2. The Natural Color System

Hue circle and hue partition

The NCS adopts the traditional form of a circle for the variations in hue. The NCS deals with four separate scales, namely one variation between yellow and red, one between red and blue, one between blue and green, and one between green and yellow. For each hue the variation of the visual content can be illustrated as a bi-polar diagram and triangular membership functions as in Fig.1. The end point of one scale is identical with the starting point of the next scale.

In Fig.2, the partition into four chromatic elementary colors is given directly by the opponent color theory as the positions of these colors on the circle (namely, as end points of diameters). This illustrates graphically the mutual exclusiveness of the attributes of opposite colors. The arc of each circle quadrant, which designates all the imaginary maximal hues of this hue scale, is divided into 100 equal steps. For example, a color that looks equally yellowish and reddish, i.e., a color that has as much of the visual component yellowness as it has of redness, is to be marked in the middle of the arc between Y and R. According to the NCS notation for hue, this will be Y50R, which means 50 hue steps from Y to R. Consequently the notation R70B means a red-blue color situated 70/100th of the way from red to blue, that is, 70% relative bluishness (30% relative reddishness). The markings on the hue circle referred to here do not represent single colors but all imaginable colors of the particular hues, i.e., all colors with the same proportion of yellowness-redness and redness-blueness, respectively, for these two examples.

NCS color triangle

The graphical illustration of all colors of a given hue is in the NCS (as in Hering's original model) equilateral triangle, where each point represents a certain nuance. (See Fig.3.) The uni-dimensional "dimension" nuance is thus the location for all colors, of all hues, with a specific relationship between the parameters of

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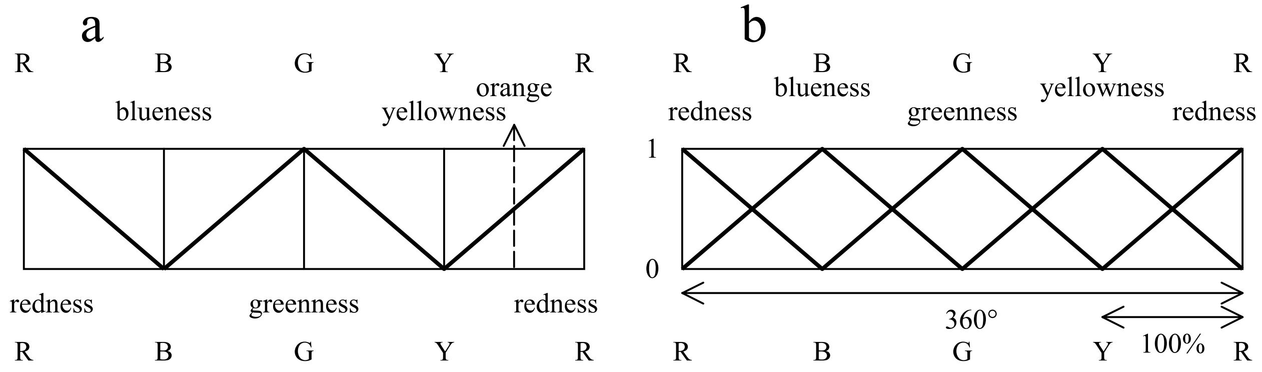

Fig.1. Hue dimension of four bi-polar scales and triangular membership function of four hues

whiteness, blackness, and chromaticness.

NCS Chromaticness

Chromaticness corresponds to the dimensional direction of Munsell's Chroma (in HVC system), but not to the scaling. Chroma is an open-ended scale, which is defined by color samples. On the other hand, NCS chromaticness has, as do all the NCS parameters, two end points and the scale between these is divided into 100 equal steps defined as "resemblance to the reference points." Chromaticness in the color triangle is the resemblance (in %) to the maximal color," by which is meant the color with the given hue, which is devoid both of whiteness and blackness. In the NCS color triangle, the corner point C in Fig.3 represents this maximal color.

NCS whiteness and blackness

The other corner points W (white) and S (black) represent correspondingly the imaginable elementary colors white and black according to Hering's phenomenological color model. The variable whiteness (w) is defined as the degree of resemblance to the imagination of the elementary white (W), and the blackness (s) is defined as the degree of resemblance to the imagination of the elementary black (S). The scaling is, as mentioned, an equispacing in 100 steps. (See Fig.3.)

NCS color space

If we now combine the hue circle with the color triangles for each of all possible hues we get the three dimensional NCS color solid. (See Fig.2.) Each point within it thus represents the perception of a surface color whose relative resemblance to one or two chromaic elementary colors and to white and black is given by the relative distances to these points in the color solid [3].

3. Methods and Results

Figure 4 illustrates input fuzzy sets of triangular pyramid, fuzzy input, and crisp output on the NCS color

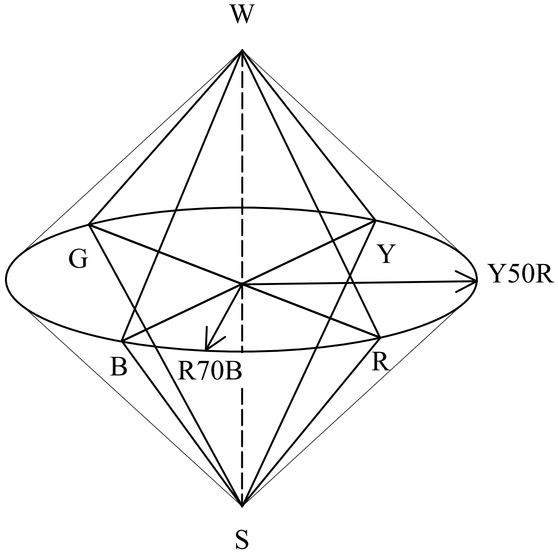

Fig.2. NCS color space and color circle

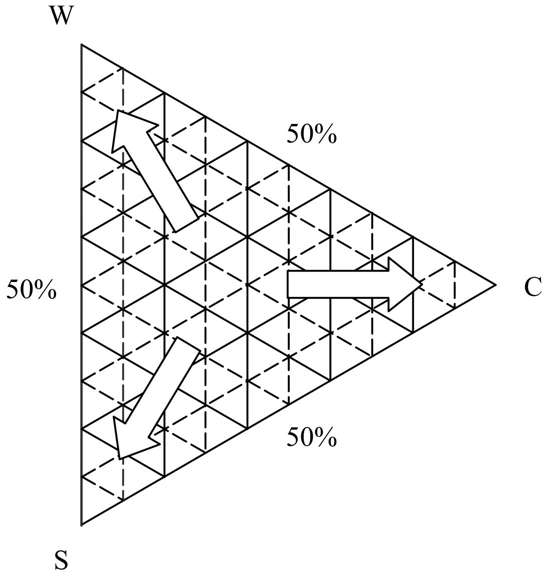

Fig.3. NCS color triangle

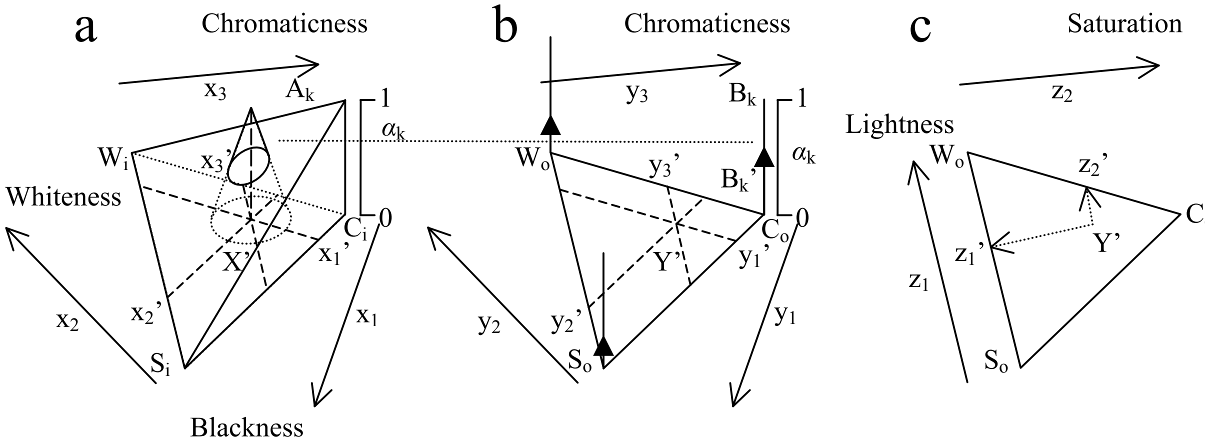

Fig.4. Fuzzy system using the membership function of a triangular pyramid on the NCS color triangle, and the HLS tone plane

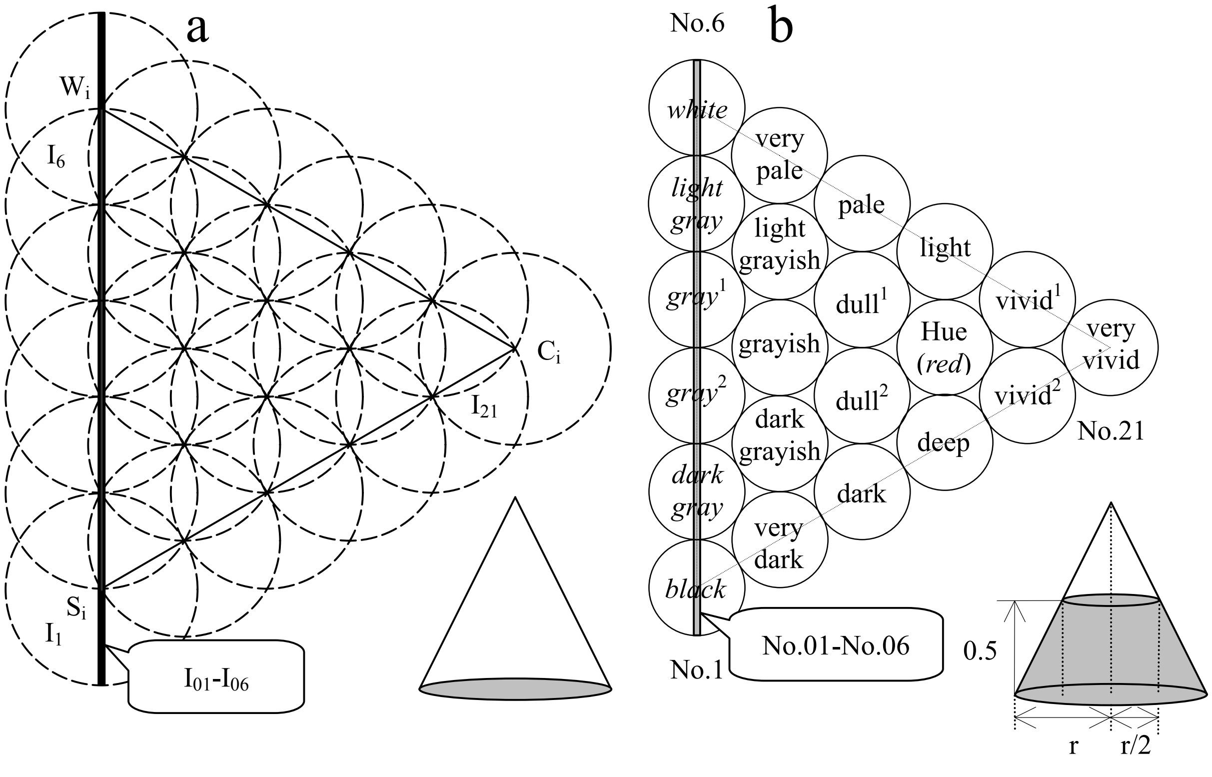

Fig.5. Fuzzy inputs on the NCS color triangle and top areas of 0.5 level-sets with color names

triangle, and crisp output on the HLS tone plane. The fuzzy rules are as follows. (See also Figs.4 and 6.)

R^1 : IF X is A_1 THEN Y is B_1
R^2 : IF X is A_2 THEN Y is B_2
R^3 : IF X is A_3 THEN Y is B_3

Rules R^k: if X is A_k , then Y is B_k (k = 1, 2, 3) where k is the rule number, A_k is a fuzzy set of input, and B_k is a crisp set of output. X=(x_1, x_2, x_3) are input parameters (variable), and Y=(y_1, y_2, y_3) are output parameters; X and Y are fixed on these parameters of NCS. A fuzzy set A_k of input shows triangular pyramid form at corner points S_i, W_i, and C_i, and a crisp set B_k of output of rule R^k is shown at corner points S_o, W_o, or C_o (a fuzzy set B_k' with arrow) on the color triangular, and output is B_k if the input is A_k.

The fuzzy inference method is as follows. Let the input be x_1=x_1', x_2=x_2', and x_3=x_3'. *i*: the input of rule R^k, grade α_k=A_k(X'), where k=1, 2, 3. *ii*: the output of rule R^k, and the α_k level-set is shown as a vertical filled allow. *iii*: B_k'=$\alpha_k B_k$, where B_k is crisp set in Fig.4b (center). *iv*: the complete inference result B' of rules R^1, R^2, and R^3.

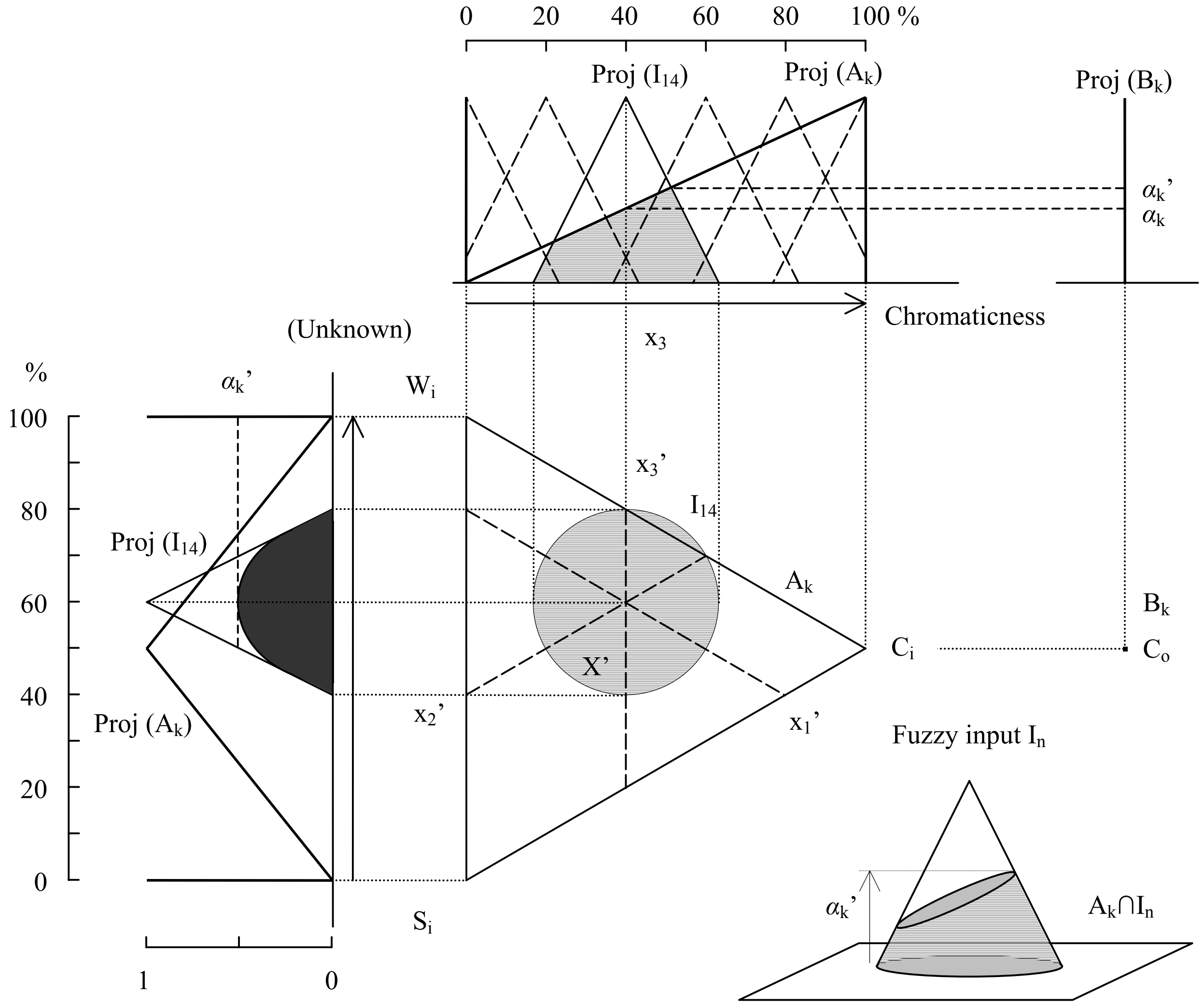

Fig.6. Membership functions of triangular pyramid on the NCS color triangle and one of 21 conical fuzzy inputs (vague chromatic colors)

$$B' = \alpha_1 B_1 \cup \alpha_2 B_2 \cup \alpha_3 B_3 = B_1' \cup B_2' \cup B_3'$$

The output parameter, $Y'=(y_1', y_2', y_3')$ corresponds to a coordinate of the central axis of the membership function of B', that is a de-fuzzification. Also, in Fig.4c (right), $Y'=(z_1', z_2')$ corresponds to a coordinate of the HLS system. z_1' (on the lightness axis) is calculated from y_1' and y_2', and z_2' (on the saturation axis) is equal to y_3'.

Figure 5a (left) illustrates fuzzy inputs (I_1-I_{21}) on the NCS color triangle. The fuzzy inputs are formed by conical membership functions and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles. The fuzzy inputs (I_{01}-I_{06}) are formed by a triangular membership function on the W_i-S_i line corresponding to the achromatic colors of the HLS system.

Figure 5b (right) illustrates arranged modifiers corresponding to conical membership functions of Fig.5a, where a modifier is shown by an area enclosed by a circle (the top of the 0.5 level-set, as shown at bottom-right). The color names and tone modifiers are the same as those used under JIS [1], [2]. Points of difference are as follows. Two membership functions are used in constructing 'grays' of achromatic colors and 'dulls' and 'vivids' of chromatic colors. In addition, 'very vivid' is added as a modifier to denote the highest chroma [4], because these modifiers occupy a large region on the plane. In this case, for instance, dull^1 and dull^2 mean lighter dull and darker dull, respectively. *Reddish* achromatic colors are included as achromatic colors [5]-[8]. Namely, cones (No.1-No.6) include triangles (No.01-No.06) on the W_i-S_i line. These color names are from the fuzzy color-naming system described in a previous work [7], [8].

Chromatic colors

Figure 6 illustrates one of 21 conical fuzzy inputs (I_1-I_{21}) on the NCS color triangle. The triangular membership function Proj(I_{14}) on the chromaticness axis

Table 1. Inference results for crisp inputs of chromatic colors

No.	Crisp input			Grade for crisp input			Inference output			HLS output	
	x_1’	x_2’	x_3’	α_1	α_2	α_3	y_1’	y_2’	y_3’	z_1’	z_2’
1	100	0	0	1.00	0.00	0.00	100.0	0.0	0.0	0.0	0.0
2	80	20	0	0.80	0.20	0.00	80.0	20.0	0.0	20.0	0.0
3	60	40	0	0.60	0.40	0.00	60.0	40.0	0.0	40.0	0.0
4	40	60	0	0.40	0.60	0.00	40.0	60.0	0.0	60.0	0.0
5	20	80	0	0.20	0.80	0.00	20.0	80.0	0.0	80.0	0.0
6	0	100	0	0.00	1.00	0.00	0.0	100.0	0.0	100.0	0.0
7	80	0	20	0.80	0.00	0.20	80.0	0.0	20.0	10.0	20.0
8	60	20	20	0.60	0.20	0.20	60.0	20.0	20.0	30.0	20.0
9	40	40	20	0.40	0.40	0.20	40.0	40.0	20.0	50.0	20.0
10	20	60	20	0.20	0.60	0.20	20.0	60.0	20.0	70.0	20.0
11	0	80	20	0.00	0.80	0.20	0.0	80.0	20.0	90.0	20.0
12	60	0	40	0.60	0.00	0.40	60.0	0.0	40.0	20.0	40.0
13	40	20	40	0.40	0.20	0.40	40.0	20.0	40.0	40.0	40.0
$\boxed{14}$	20	40	40	0.20	0.40	0.40	$\underline{20.0}$	$\underline{40.0}$	$\underline{40.0}$	$\underline{60.0}$	$\underline{40.0}$
$\boxed{15}$	0	60	40	0.00	0.60	0.40	$\underline{0.0}$	$\underline{60.0}$	$\underline{40.0}$	$\underline{80.0}$	$\underline{40.0}$
16	40	0	60	0.40	0.00	0.60	40.0	0.0	60.0	30.0	60.0
17	20	20	60	0.20	0.20	0.60	20.0	20.0	60.0	50.0	60.0
$\boxed{18}$	0	40	60	0.00	0.40	0.60	$\underline{0.0}$	$\underline{40.0}$	$\underline{60.0}$	$\underline{70.0}$	$\underline{60.0}$
19	20	0	80	0.20	0.00	0.80	20.0	0.0	80.0	40.0	80.0
20	0	20	80	0.00	0.20	0.80	0.0	20.0	80.0	60.0	80.0
21	0	0	100	0.00	0.00	1.00	0.0	0.0	100.0	50.0	100.0

is one of six projections of 21 fuzzy inputs (I_1-I_{21}) by the rays from the lower part, and the triangular membership function Proj(I_{14}) on the unknown axis in the NCS (corresponding to lightness on the HLS system) is one of 11 projections of 21 fuzzy inputs (I_1-I_{21}) by the rays from the right side. However, the latter projection does not work at all in this case.

An intersection of input fuzzy set A_k for fuzzy input I_n is $A_k \cap I_n$. (See the shaded area at the bottom-right of Fig.6.) Grade α_k’= height ($A_k \cap I_n$). If the input is crisp, α_k’ changes α_k. C_o is the new maximum chromaticness. Proj(B_k) is a projection of an output crisp set at the corner point C_o (See Fig.4b, center).

What happens if a vague color is input into the NCS? The system considered in this study can translate input data X of vague color to output data Y of simple color on the NCS color triangle when the hue is fixed as red. The fuzzy input (No.14) on the NCS is constructed by the center X’= (x_1’, x_2’, x_3’) = (20, 40, 40) in % and the radius r =23% of the basal plane (circle) of the cone indicated vagueness.

Table 1 shows the fuzzy inference results for 21 crisp inputs (x_1’, x_2’, x_3’). The grades (α_1, α_2, α_3), inference outputs (y_1’, y_2’, y_3’), and HLS output Y’= (z_1’, z_2’) were calculated. The relationship between

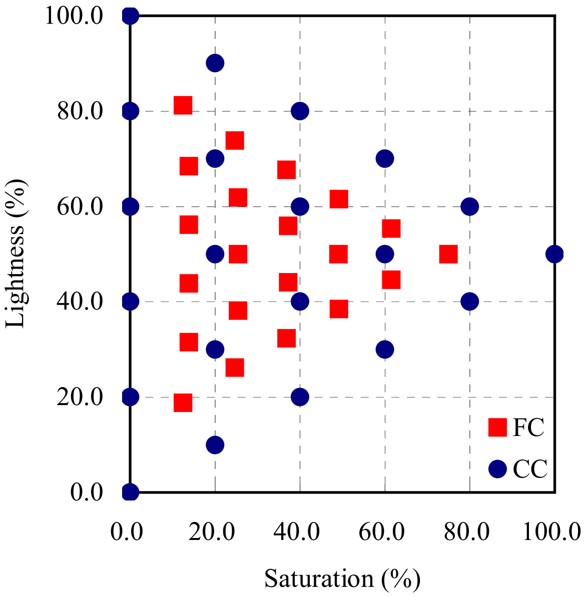

Fig.7. Fuzzy inference outputs for crisp (circle) and fuzzy (square) inputs on the HLS tone plane corresponding to Fig.4c

Table 2. Inference results for fuzzy inputs of chromatic colors

No.	Center of fuzzy input			Grade for fuzzy input			Inference output			HLS output	
	x_1'	x_2'	x_3'	α_1'	α_2'	α_3'	y_1'	y_2'	y_3'	z_1'	z_2'
1	100	0	0	1.00	0.17	0.17	75.0	12.5	12.5	18.8	12.5
2	80	20	0	0.83	0.33	0.19	61.5	24.6	13.9	31.5	13.9
3	60	40	0	0.67	0.50	0.19	49.2	36.9	13.9	43.8	13.9
4	40	60	0	0.50	0.67	0.19	36.9	49.2	13.9	56.2	13.9
5	20	80	0	0.33	0.83	0.19	24.6	61.5	13.9	68.5	13.9
6	0	100	0	0.17	1.00	0.17	12.5	75.0	12.5	81.3	12.5
7	80	0	20	0.83	0.19	0.33	61.5	13.9	24.6	26.2	24.6
8	60	20	20	0.67	0.35	0.35	49.1	25.5	25.5	38.2	25.5
9	40	40	20	0.51	0.51	0.35	37.3	37.3	25.5	50.0	25.5
10	20	60	20	0.35	0.67	0.35	25.5	49.1	25.5	61.8	25.5
11	0	80	20	0.19	0.83	0.33	13.9	61.5	24.6	73.8	24.6
12	60	0	40	0.67	0.19	0.50	49.2	13.9	36.9	32.3	36.9
13	40	20	40	0.51	0.35	0.51	37.3	25.5	37.3	44.1	37.3
$\boxed{14}$	20	40	40	0.35	0.51	0.51	$\underline{25.5}$	$\underline{37.3}$	$\underline{37.3}$	$\underline{55.9}$	$\underline{37.3}$
$\boxed{15}$	0	60	40	0.19	0.67	0.50	$\underline{13.9}$	$\underline{49.2}$	$\underline{36.9}$	$\underline{67.7}$	$\underline{36.9}$
16	40	0	60	0.50	0.19	0.67	36.9	13.9	49.2	38.5	49.2
17	20	20	60	0.35	0.35	0.67	25.5	25.5	49.1	50.0	49.1
$\boxed{18}$	0	40	60	0.19	0.50	0.67	$\underline{13.9}$	$\underline{36.9}$	$\underline{49.2}$	$\underline{61.5}$	$\underline{49.2}$
19	20	0	80	0.33	0.19	0.83	24.6	13.9	61.5	44.6	61.5
20	0	20	80	0.19	0.33	0.83	13.9	24.6	61.5	55.4	61.5
21	0	0	100	0.17	0.17	1.00	12.5	12.5	75.0	50.0	75.0

inputs, grades, and outputs is proportional. If hue is fixed to red, chromatic color names are No.1: *reddish* black, No.2: dark *reddish* gray, No.3: *reddish* gray^2, No.4: *reddish* gray^1, No.5: light *reddish* gray, No.6: *reddish* white, No.7: very dark, No.8: dark grayish, No.9: grayish, No.10: light grayish, No.11: very pale, No.12: dark, No.13: dull^2, No.14: dull^1, No.15: pale, No.16: deep, No.17: hue (red), No.18: light, No.19: vivid^2, No.20: vivid^1, and No.21: very vivid. These numbers correspond to the suffix of fuzzy input I.

Table 2 shows the fuzzy inference results for 21 fuzzy inputs. The fuzzy inputs on the NCS are constructed by the centers (x_1', x_2', x_3') in the same manner as the crisp inputs in Table 1 and the radius (r =23%) of the basal plane (circle) of the cone indicated vagueness. (See Fig.5.) The inputs and outputs are not proportional, which indicates the nonlinear information processing in this fuzzy system.

Figure 7 illustrates a relationship between the lightness z_1 and the saturation z_2 obtained from data (z_1', z_2') in Table 1 and 2. Squares (FC) indicate outputs for fuzzy inputs of chromatic colors (including reddish achromatic colors), circles (CC) indicate outputs for crisp inputs of chromatic and achromatic colors corresponding to Fig.4c. The outputs (circle points) are

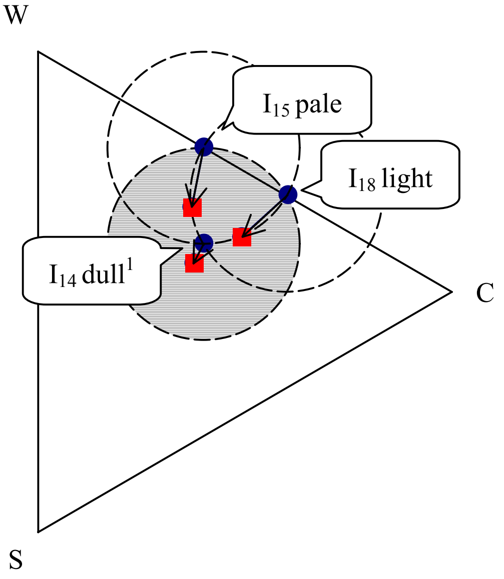

Fig.8. Relationship between lighter dull (dull^1), pale, and light on the NCS color triangle corresponding to Fig.4b

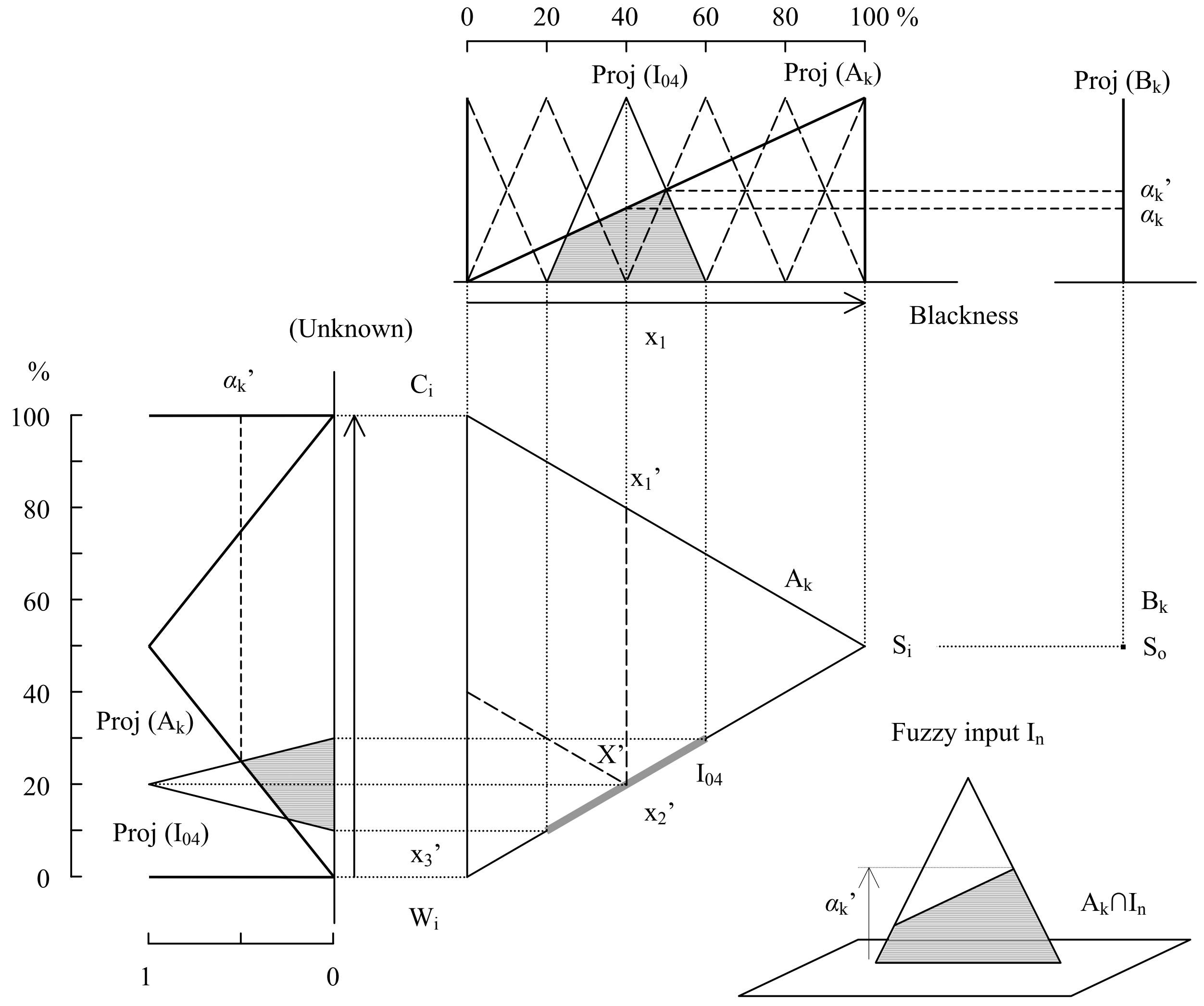

Fig.9. Membership functions of triangular pyramid on the NCS color triangle and one of triangular fuzzy inputs (vague achromatic colors)

the same coordinates for the input points in the NCS. The outputs (squares points) for fuzzy inputs are grouped at the center of the tone triangle. The points are clearly different. Vague color inputs to the NCS (Fig.4a), the system outputs crisp color in the NCS (Fig.4b), and also outputs crisp color in the HLS system (Fig.4c). The coordinates of crisp inputs are the same as those of crisp outputs.

Figure 8 illustrates the differences between crisp and fuzzy inputs. Large circles show fuzzy inputs I_{14}, I_{15}, and I_{18}, which are JIS-like tone modifiers: “dull^1” (lighter dull), “pale”, and “light”, respectively. (See squared numbers and underlined values in Table 2.) Three arrows show changing directions from outputs for crisp to those for fuzzy (large circle). For I_{14}: dull^1, the arrow indicates a change toward blackness S. Output (square) for fuzzy input is dislocated from the center of fuzzy input (shaded large circle). On the other hand, output for crisp input is located at the center. Crisp input and its inference output are in the same coordinate. For “pale” and “light”, each output for fuzzy inputs is distant from the centers of large circles and is quite close to the “dull^1” centers, rather than the other centers. This implies that “pale” and “light” change to “dull^1”, if such vague colors are input into the fuzzy system.

When an inference output falls within the shaded circular area, closeness can be obtained simply from the distance between the center of the circle (crisp input) and the inference output. Determining closeness β is equivalent to calculating the ratio $(r - d) / r$, where r is the radius of the circle, and d is the distance from the fuzzy input as the center of the shaded large circle to the inference output. For I_{14} the distance: d=4.9 (from the center of fuzzy input to the inference output of I_{14}) and closeness: β=0.79. For I_{15} the distance: d=8.3 (from the center of fuzzy input of I_{14} to the inference output of I_{15}) and closeness: β=0.64. And for I_{18}: d=9.3 and β=0.60. That is, the vagueness of dull^1 shows a closeness of 0.79 in dull^1, the vagueness of pale shows a closeness of 0.64 in dull^1, and vagueness of light shows a closeness of 0.60 in dull^1. This means that a closeness of more than 0.50 indicates the modifier dull^1 in this case (see

Table 3. Inference results for crisp inputs of achromatic colors

No.	Crisp input			Grade for crisp input			Inference output			HLS output	
	x_1’	x_2’	x_3’	α_1	α_2	α_3	y_1’	y_2’	y_3’	z_1’	z_2’
01	100	0	0	1.00	0.00	0.00	100.0	0.0	0.0	0.0	0.0
02	80	20	0	0.80	0.20	0.00	80.0	20.0	0.0	20.0	0.0
03	60	40	0	0.60	0.40	0.00	60.0	40.0	0.0	40.0	0.0
04	40	60	0	0.40	0.60	0.00	40.0	60.0	0.0	60.0	0.0
05	20	80	0	0.20	0.80	0.00	20.0	80.0	0.0	80.0	0.0
06	0	100	0	0.00	1.00	0.00	0.0	100.0	0.0	100.0	0.0

Table 4. Inference results for fuzzy inputs of achromatic colors

No.	Center of fuzzy input			Grade for fuzzy input			Inference output			HLS output	
	x_1’	x_2’	x_3’	α_1’	α_2’	α_3’	y_1’	y_2’	y_3’	z_1’	z_2’
01	100	0	0	1.00	0.17	0.00	85.7	14.3	0.0	14.3	0.0
02	80	20	0	0.83	0.33	0.00	71.4	28.6	0.0	28.6	0.0
03	60	40	0	0.67	0.50	0.00	57.1	42.9	0.0	42.9	0.0
04	40	60	0	0.50	0.67	0.00	42.9	57.1	0.0	57.1	0.0
05	20	80	0	0.33	0.83	0.00	28.6	71.4	0.0	71.4	0.0
06	0	100	0	0.17	1.00	0.00	14.3	85.7	0.0	85.7	0.0

Fig.5b). Therefore, dull^1 with vagueness can be referred to as widely dull, pale and light with vagueness can also be referred to as dull.

Achromatic colors

Figure 9 illustrates fuzzy inputs (I_{01}-I_{06}) on the NCS color triangle. The fuzzy inputs are formed by triangular membership functions, which are made to mutually overlap. The triangular membership functions are included in the conical membership function. The triangular membership function Proj(I_{04}) on the unknown axis (not an NCS parameter) is one of six projections of six fuzzy inputs (I_{01}-I_{06}) by the rays from the right side, and the triangular membership function Proj(I_{04}) on the blackness axis is one of six projections of those by the rays from the lower part.

An intersection of A_k for fuzzy sets I_n of input is $A_k \cap I_n$. (See the shaded area in Fig.9 bottom-right.) Grade α_k’= height ($A_k \cap I_n$). If the input is crisp, α_k’ changes to α_k. S_o is the new blackness. Proj(B_k) is a projection of the output fuzzy set at the corner point S_o (Fig.4b, center).

In the present study, for vague achromatic color inputs to the NCS, this system can translate input data X of vague color to output data Y of simple color on the NCS color triangle. The fuzzy input (No.04) on the NCS is constructed by the center X’= (x_1’, x_2’, x_3’) = (40, 60, 0) as percentages and the basal line of the triangle indicating vagueness is $2 \cdot r$ =46% long.

Table 3 shows the fuzzy inference results for six crisp inputs (x_1’, x_2’, x_3’). The grades (α_1, α_2, α_3), inference outputs (y_1’, y_2’, y_3’), and HLS output Y’= (z_1’, z_2’) were calculated. The relationship between inputs, grades, and outputs are proportional. The achromatic color names are No.01: black, No.02: dark gray, No.03: gray^2, No.04: gray^1, No.05: light gray, and No.06: white. These numbers correspond to the suffixes of fuzzy input I.

Table 4 shows the fuzzy inference results for six fuzzy inputs. The fuzzy inputs on the NCS are constructed by the centers (x_1’, x_2’, x_3’) in the same manner as in Table 3 and the basal line of the triangle-indicated vagueness is 46% of the total length. The inputs and outputs are not proportional, which indicates nonlinear information processing in this fuzzy system.

Figure 10 illustrates the relationship between the lightness z_1 and the saturation z_2 obtained from data (z_1’, z_2’) in Tables 1, 2, 3, and 4. Filled squares (FC) are outputs for fuzzy inputs of chromatic colors, and filled circles (CC) are outputs for crisp inputs of chromatic colors already shown in Fig.7. In addition, open squares (FA) are outputs for fuzzy inputs of achromatic colors, and open circles (CA) are outputs for crisp inputs of achromatic colors. Circles indicate the corresponding coordinate for the input points in the NCS. Squares indicate the centers of tone triangles. That is, circles move toward square points using fuzzy inference.

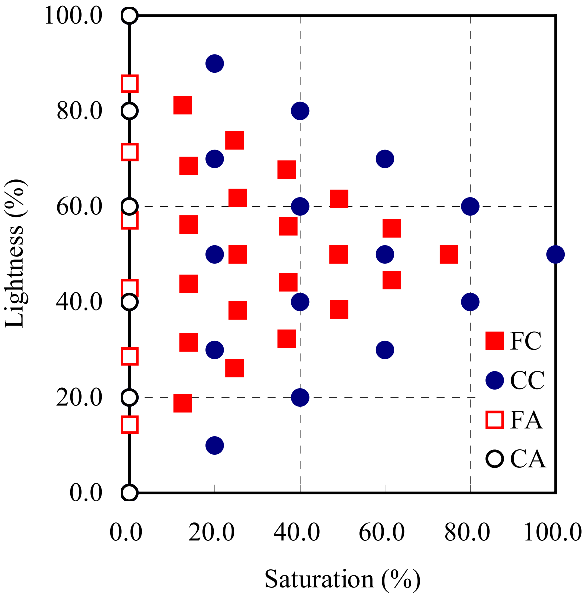

Fig.10. Fuzzy inference results for crisp (circle) and fuzzy (square) inputs on the HLS tone plane

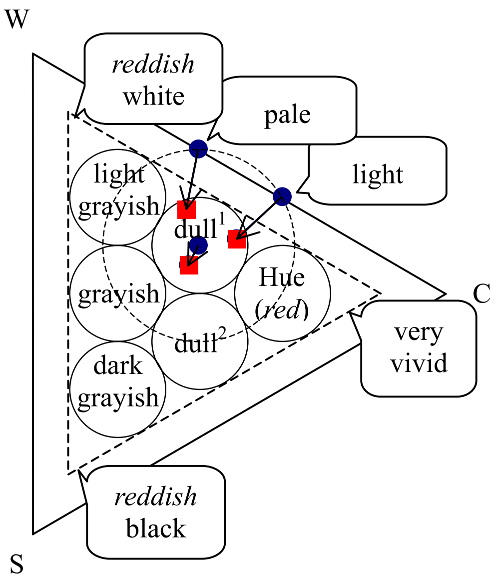

Fig.11. Schematic gathering effects of outputs for conical fuzzy input

The relationship between crisp inputs (centers of large circles), conical fuzzy inputs (large circles), top areas of 0.5 level-sets of each modifier (medium circles), and fuzzy inference outputs (circles and squares) on the NCS color triangle is examined.

Figure 11 shows a schematic illustration, which focuses on one dull^1 (lighter dull) area. The centers of modifiers “pale” and “light” as the coordinates of crisp inputs move to the area (middle circle) of “dull^1”, because the vagueness of two modifiers is inferred. The vagueness (large circle) of dull^1 is also inferred, and then center point moves to the *blackness* direction. The distances between circles and squares (indicated arrows) are shown in Table 5.

Table 5 shows distances between centers of six specific vague color inputs (*red* hue, dull^1, dull^2, light grayish, grayish, and dark grayish) and all fuzzy inference outputs. Values enclosed by squares indicate the closeness of neighboring colors. The squared values show distances between the centers of vague color inputs and their fuzzy inference outputs, and the underlined values show a few closer distances between the centers of vague color inputs and other fuzzy inference outputs. These exist in the top areas of the 0.5 level-sets of each modifier.

4. Conclusions

The present paper proposes a fuzzy system that, for a known hue name (e.g., red), can extract crisp outputs of the NCS (which is available for use in fuzzy set theory) and HLS system (which is easy to show via graphs). It is difficult to construct such a fuzzy system on the tone plane in the HLS system, because the membership function of the lightness is quite complicated on the tone triangle. The system also extracts, in a simple manner, the membership grades from the projection of a conical membership function of a vague color input. Three parameters associated with respective grades indicate vague colors and output the center of gravity as a crisp color value although the NCS does not have a ‘lightness’ attribute.

In the near future, this system will help to ensure important color information (e.g. vagueness, shadings of color) of goods and arts by reducing the confusion between colors often experienced by people.

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Table 5. Distances between centers of six vague color inputs and fuzzy inference outputs

No.	Color names	Distances from No. n					
		Hue (*red*)	dull^1	dull^2	light grayish	grayish	dark grayish
01	*black*	-	-	-	59.2	40.9	25.4
02	*dark gray*	-	-	-	46.0	29.3	20.1
03	*gray*^{2}	-	-	-	33.7	21.2	23.8
04	*gray*^{1}	-	-	-	23.8	21.2	33.7
05	*light gray*	-	-	-	20.1	29.3	46.0
06	*white*	-	-	-	25.4	40.9	59.2
1	*reddish* black	56.9	49.6	34.8	51.8	32.1	13.5
2	dark *reddish* gray	49.7	38.6	27.5	38.9	19.4	$\underline{6.3}$
3	*reddish* gray^2	46.5	30.7	26.4	26.9	$\underline{8.7}$	15.1
4	*reddish* gray^1	46.5	26.4	30.7	15.1	$\underline{8.7}$	26.9
5	light *reddish* gray	49.7	27.5	38.6	$\underline{6.3}$	19.4	38.9
6	*reddish* white	56.9	34.8	49.6	13.5	32.1	51.8
7	very dark	42.7	37.2	20.7	44.1	24.3	$\underline{6.0}$
$\boxed{8}$	dark grayish	36.5	26.2	14.7	32.3	13.0	$\boxed{9.8}$
$\boxed{9}$	grayish	34.5	17.7	17.7	20.7	$\boxed{5.5}$	20.7
$\boxed{10}$	light grayish	36.5	14.7	26.2	$\boxed{9.8}$	13.0	32.3
11	very pale	42.7	20.7	37.2	$\underline{6.0}$	24.3	44.1
12	dark	29.1	27.8	$\underline{8.3}$	41.3	24.5	17.1
$\boxed{13}$	dull^2	23.5	16.1	$\boxed{4.9}$	31.1	18.3	22.3
$\boxed{14}$	dull^1	23.5	$\boxed{4.9}$	16.1	22.3	18.3	31.1
15	pale	29.1	$\underline{8.3}$	27.8	17.1	24.5	41.3
16	deep	15.8	23.4	$\underline{9.3}$	43.0	31.4	30.4
$\boxed{17}$	Hue (*red*)	$\boxed{10.9}$	13.5	13.5	35.3	29.1	35.3
18	light	15.8	$\underline{9.3}$	23.4	30.4	31.4	43.0
19	vivid^2	$\underline{5.6}$	26.4	22.0	48.7	41.9	44.0
20	vivid^1	$\underline{5.6}$	22.0	26.4	44.0	41.9	48.7
21	very vivid	15.0	36.4	36.4	58.5	55.0	58.5

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