

Natural Color System Using Fuzzy Set Theoretical Approach

Naotoshi Sugano
Tamagawa University

6-1-1 Tamagawagakuen, Machida, Tokyo 194-6810, JAPAN
E-mail: sugano@eng.tamagawa.ac.jp

Abstract: The present study considered a fuzzy natural color system (NCS) in which triangular pyramid membership functions are constructed on the NCS color triangle. This system can process a fuzzy input to an NCS system and output to a center of gravity of three weights associated with respective grades. Triangular membership functions are applied to the hue, and triangular pyramid membership functions are applied to the NCS color triangle relationship. By treating three membership functions of blackness, whiteness, and chromaticness on the NCS color triangle, we can easily obtain a target color as a center of gravity of the output fuzzy set. We will show herein the difference between crisp inputs and fuzzy inputs.

1. Introduction

The NCS (Natural Color System) color descriptive system and atlas were developed in Sweden [3]. In the NCS system, a method for obtaining hue expressions has been reported by Sivik (1997). However, a technique for obtaining expressions of the NCS color triangle using the fuzzy set theoretical method has not been studied.

In this study, fuzzy inputs of conical membership functions are used on the NCS color triangle corresponding to the HLS tone plane [5]-[8]. Such a system will help us to determine the average color value as a center of gravity of attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

2. The NCS system

NCS color triangle

The graphical illustration of all colors of a given hue is in the NCS (as in Hering's original model) equilateral triangle, where each point represents a certain nuance. (See Fig.1.) The uni-dimensional "dimension" nuance is thus the location for all colors, of all hues, with a specific relationship between the parameters of whiteness, blackness, and chromaticness.

NCS chromaticness

Chromaticness corresponds to the dimensional direction of Munsell's Chroma (in HVC system), but not to the scaling. Chroma is an open-ended scale, which is defined by color samples. On the other hand, NCS chromaticness has, as do all the NCS parameters, two end points and the scale between these is divided into 100 equal steps defined as "resemblance to the reference points." Chromaticness in the color triangle is the resemblance (in %) to the maximal color," by which is meant the color with the given hue, which is devoid both of whiteness and blackness. In the NCS color

triangle, the corner point C in Fig.1 represents this maximal color.

NCS whiteness and blackness

The other corner points W (white) and S (black) represent correspondingly the imaginable elementary colors white and black according to Hering's phenomenological color model. The variable whiteness (w) is defined as the degree of resemblance to the imagination of the elementary white (W), and the blackness (s) is defined as the degree of resemblance to the imagination of the elementary black (S). The scaling is, as mentioned, an equispacing in 100 steps. (See Fig.1.)

3. Methods and Results

Figure 2 illustrates input fuzzy sets of triangular pyramid, fuzzy input, and crisp output on the NCS

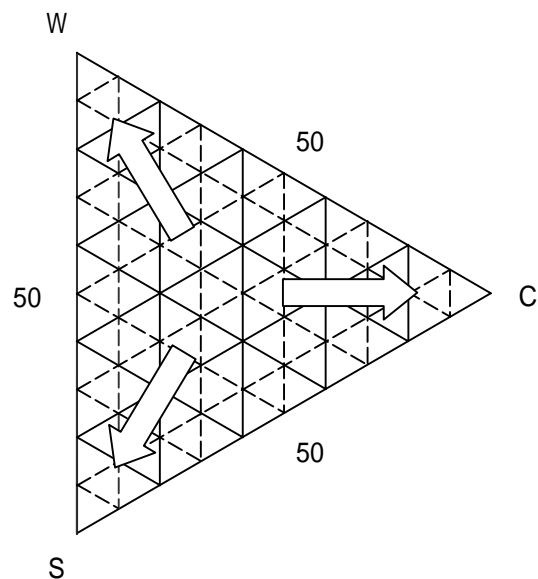


Fig.1. NCS color triangle

color triangle, and crisp output on the HLS tone plane. The fuzzy rules are as follows. (See also Figs.2 and 4.)

- R^1 : IF X is A_1 THEN Y is B_1
- R^2 : IF X is A_2 THEN Y is B_2
- R^3 : IF X is A_3 THEN Y is B_3

Rules R^k : if X is A_k , then Y is B_k ($k = 1, 2, 3$) where k is the rule number, A_k is a fuzzy set of input, and B_k is a crisp set of output. $X=(x_1, x_2, x_3)$ are input parameters (variable), and $Y=(y_1, y_2, y_3)$ are output parameters; X and Y are fixed on these parameters of NCS. A fuzzy set A_k of input shows triangular pyramid form at corner points $S_i, W_i,$ and $C_i,$ and a crisp set B_k of output of rule

R^k is shown at corner points $S_o, W_o,$ or C_o (a fuzzy set B'_k with arrow) on the color triangular, and output is B_k if the input is A_k .

The fuzzy inference method is as follows. Let the input be $x_1=x_1', x_2=x_2',$ and $x_3=x_3'$. *i*: the input of rule R^k , grade $\alpha_k = A_k(X')$, where $k=1, 2, 3$. *ii*: the output of rule R^k , and the α_k level-set is shown as a vertical filled allow. *iii*: $B'_k = \alpha_k B_k$, where B_k is fuzzy set in Fig.2b (center). *iv*: the complete inference result B' of rules $R^1, R^2,$ and R^3 .

$$B' = \alpha_1 B_1 \cup \alpha_2 B_2 \cup \alpha_3 B_3 = B_1' \cup B_2' \cup B_3'$$

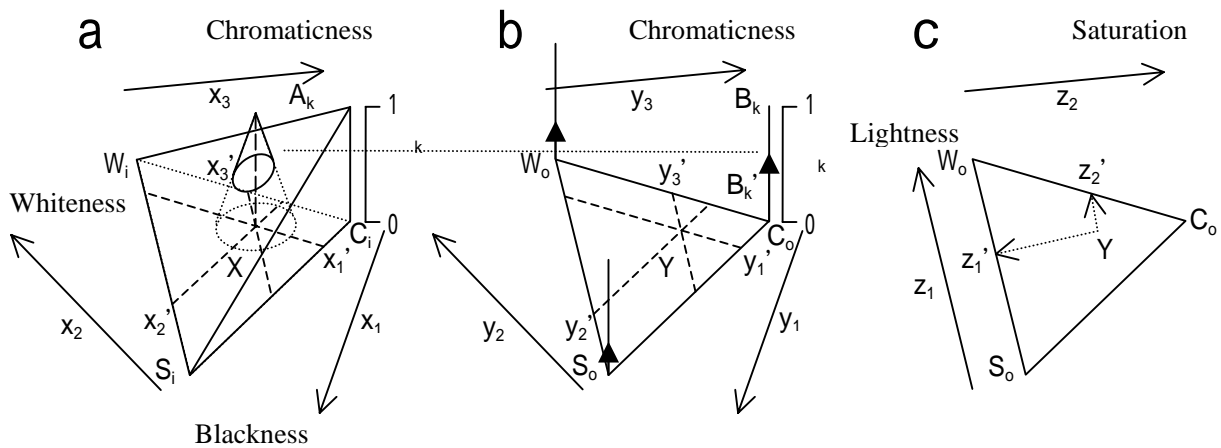


Fig.2. Fuzzy system using the membership function of a triangular pyramid on the NCS color triangle, and the HLS tone plane

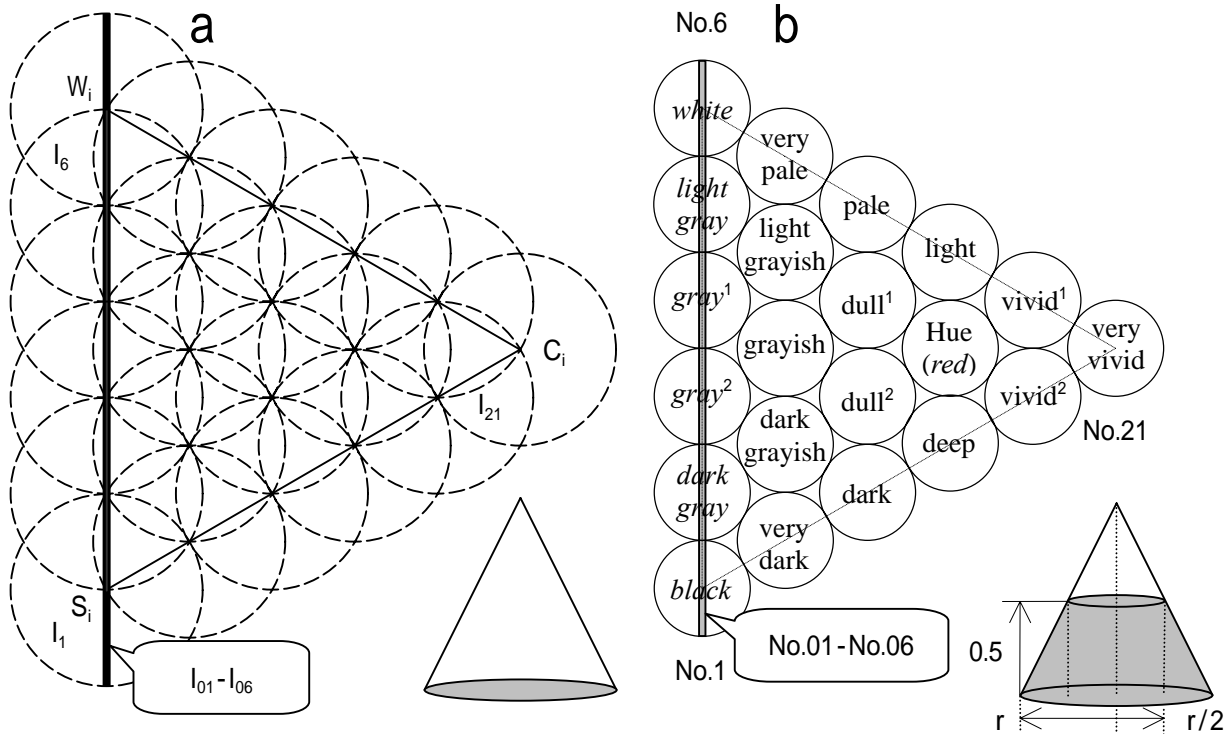


Fig.3. Fuzzy inputs on the NCS color triangle and top areas of 0.5 level-sets with color names

The output parameter, $Y'=(y_1', y_2', y_3')$ corresponds to a coordinate of the central axis of the membership function of B' , that is a de-fuzzification. Also, in Fig.2c (right), $Y'=(z_1', z_2')$ corresponds to a coordinate of the HLS system. z_1' (on the lightness axis) is calculated from y_1' and y_2' , and z_2' (on the saturation axis) is equal to y_3' .

Figure 3a (left) illustrates fuzzy inputs (I_1 - I_{21}) on the NCS color triangle. The fuzzy inputs are formed by conical membership functions and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles. The fuzzy inputs (I_{01} - I_{06}) are formed by a triangular membership function on the W_i - S_i line corresponding to the achromatic colors of the HLS system.

Figure 3b (right) illustrates arranged modifiers corresponding to conical membership functions of Fig.3a, where a modifier is shown by an area enclosed by a circle (the top of the 0.5 level-set, as shown at bottom-right). The color names and tone modifiers are the same as those used under JIS [1], [2]. Points of

difference are as follows. Two membership functions are used in constructing 'grays' of achromatic colors and 'dulls' and 'vivid' of chromatic colors. In addition, 'very vivid' is added as a modifier to denote the highest chroma [4], because these modifiers occupy a large region on the plane. In this case, for instance, dull¹ and dull² mean lighter dull and darker dull, respectively. *Reddish* achromatic colors are included as achromatic colors [5]-[8]. Namely, cones (No.1-No.6) include triangles (No.01-No.06) on the W_i - S_i line. These color names are from the fuzzy color-naming system described in a previous work [7], [8].

Chromatic colors

Figure 4 illustrates one of 21 conical fuzzy inputs (I_1 - I_{21}) on the NCS color triangle. The triangular membership function $Proj(I_{14})$ on the chromaticness axis is one of six projections of 21 fuzzy inputs (I_1 - I_{21}) by the rays from the lower part, and the triangular membership function $Proj(I_{14})$ on the unknown axis in the NCS system (corresponding to lightness on the HLS system) is one of 11 projections of 21 fuzzy inputs

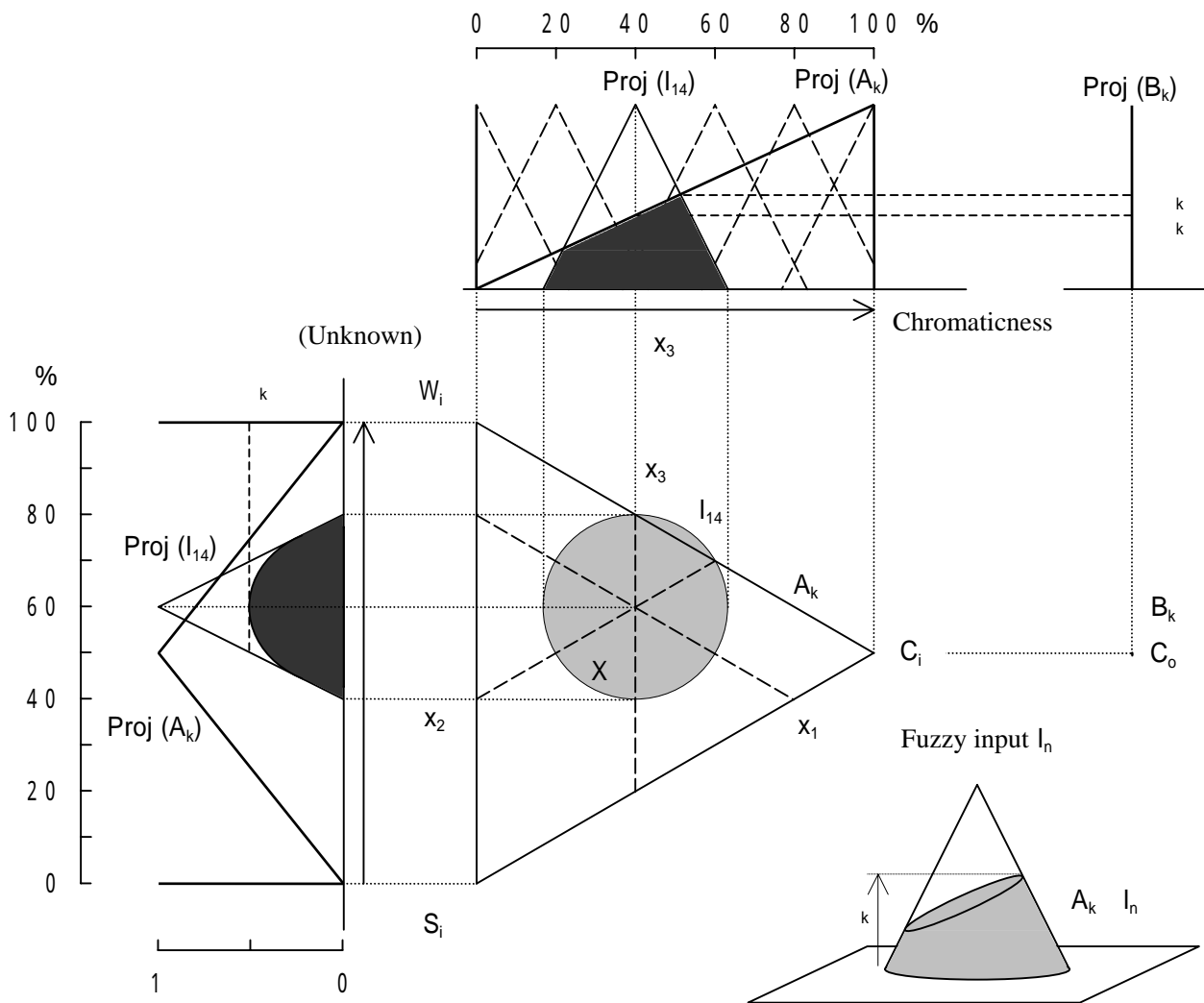


Fig.4. Membership functions of triangular pyramid on the NCS color triangle and one of 21 conical fuzzy inputs (vague chromatic colors)

Table 1. Inference results for crisp inputs of chromatic colors

No.	Crisp input			Grade for crisp input			Inference output			HLS output	
	x_1'	x_2'	x_3'	α_1	α_2	α_3	y_1'	y_2'	y_3'	z_1'	z_2'
1	100	0	0	1.00	0.00	0.00	100.0	0.0	0.0	0.0	0.0
2	80	20	0	0.80	0.20	0.00	80.0	20.0	0.0	20.0	0.0
3	60	40	0	0.60	0.40	0.00	60.0	40.0	0.0	40.0	0.0
4	40	60	0	0.40	0.60	0.00	40.0	60.0	0.0	60.0	0.0
5	20	80	0	0.20	0.80	0.00	20.0	80.0	0.0	80.0	0.0
6	0	100	0	0.00	1.00	0.00	0.0	100.0	0.0	100.0	0.0
7	80	0	20	0.80	0.00	0.20	80.0	0.0	20.0	10.0	20.0
8	60	20	20	0.60	0.20	0.20	60.0	20.0	20.0	30.0	20.0
9	40	40	20	0.40	0.40	0.20	40.0	40.0	20.0	50.0	20.0
10	20	60	20	0.20	0.60	0.20	20.0	60.0	20.0	70.0	20.0
11	0	80	20	0.00	0.80	0.20	0.0	80.0	20.0	90.0	20.0
12	60	0	40	0.60	0.00	0.40	60.0	0.0	40.0	20.0	40.0
13	40	20	40	0.40	0.20	0.40	40.0	20.0	40.0	40.0	40.0
<u>14</u>	20	40	40	0.20	0.40	0.40	<u>20.0</u>	<u>40.0</u>	<u>40.0</u>	<u>60.0</u>	<u>40.0</u>
<u>15</u>	0	60	40	0.00	0.60	0.40	<u>0.0</u>	<u>60.0</u>	<u>40.0</u>	<u>80.0</u>	<u>40.0</u>
16	40	0	60	0.40	0.00	0.60	40.0	0.0	60.0	30.0	60.0
17	20	20	60	0.20	0.20	0.60	20.0	20.0	60.0	50.0	60.0
<u>18</u>	0	40	60	0.00	0.40	0.60	<u>0.0</u>	<u>40.0</u>	<u>60.0</u>	<u>70.0</u>	<u>60.0</u>
19	20	0	80	0.20	0.00	0.80	20.0	0.0	80.0	40.0	80.0
20	0	20	80	0.00	0.20	0.80	0.0	20.0	80.0	60.0	80.0
21	0	0	100	0.00	0.00	1.00	0.0	0.0	100.0	50.0	100.0

(I_1 - I_{21}) by the rays from the right side. However, the latter projection does not work at all in this case.

An intersection of input fuzzy set A_k for fuzzy input I_n is $A_k \cap I_n$. (See the shaded area at the bottom-right of Fig.4.) Grade $\alpha_k = \text{height}(A_k \cap I_n)$. If the input is crisp, α_k changes α_k . C_0 is the new maximum chromaticness. $\text{Proj}(B_k)$ is a projection of an output crisp set at the corner point C_0 (See Fig.2b, center).

What happens if a vague color is input into the NCS system? The system considered in this study can translate input data X of vague color to output data Y of simple color on the NCS color triangle when the hue is fixed as red. The fuzzy input (No.14) on the NCS is constructed by the center $X' = (x_1', x_2', x_3') = (20, 40, 40)$ in % and the radius $r = 23\%$ of the basal plane (circle) of the cone indicated vagueness.

Table 1 shows the fuzzy inference results for 21 crisp inputs (x_1', x_2', x_3'). The grades ($\alpha_1, \alpha_2, \alpha_3$), inference outputs (y_1', y_2', y_3'), and HLS output $Y' = (z_1', z_2')$ were calculated. The relationship between inputs, grades, and outputs is proportional. If hue is fixed to red, chromatic color names are No.1: *reddish* black, No.2: dark *reddish* gray, No.3: *reddish* gray², No.4: *reddish* gray¹, No.5: light *reddish* gray, No.6: *reddish* white, No.7: very dark, No.8: dark grayish, No.9: grayish, No.10: light grayish, No.11: very pale, No.12:

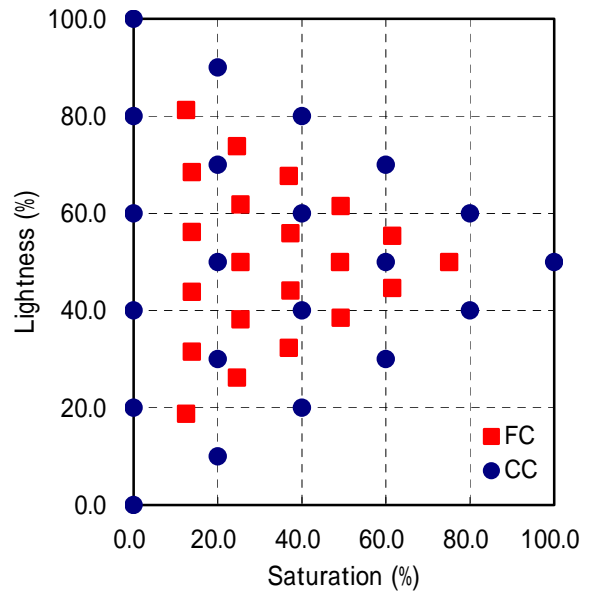


Fig.5. Fuzzy inference outputs for crisp (circle) and fuzzy (square) inputs on the HLS tone plane corresponding to Fig.2c

Table 2. Inference results for fuzzy inputs of chromatic colors

No.	Center of fuzzy input			Grade for fuzzy input			Inference output			HLS output	
	x_1'	x_2'	x_3'	a_1'	a_2'	a_3'	y_1'	y_2'	y_3'	z_1'	z_2'
1	100	0	0	1.00	0.17	0.17	75.0	12.5	12.5	18.8	12.5
2	80	20	0	0.83	0.33	0.19	61.5	24.6	13.9	31.5	13.9
3	60	40	0	0.67	0.50	0.19	49.2	36.9	13.9	43.8	13.9
4	40	60	0	0.50	0.67	0.19	36.9	49.2	13.9	56.2	13.9
5	20	80	0	0.33	0.83	0.19	24.6	61.5	13.9	68.5	13.9
6	0	100	0	0.17	1.00	0.17	12.5	75.0	12.5	81.3	12.5
7	80	0	20	0.83	0.19	0.33	61.5	13.9	24.6	26.2	24.6
8	60	20	20	0.67	0.35	0.35	49.1	25.5	25.5	38.2	25.5
9	40	40	20	0.51	0.51	0.35	37.3	37.3	25.5	50.0	25.5
10	20	60	20	0.35	0.67	0.35	25.5	49.1	25.5	61.8	25.5
11	0	80	20	0.19	0.83	0.33	13.9	61.5	24.6	73.8	24.6
12	60	0	40	0.67	0.19	0.50	49.2	13.9	36.9	32.3	36.9
13	40	20	40	0.51	0.35	0.51	37.3	25.5	37.3	44.1	37.3
<u>14</u>	20	40	40	0.35	0.51	0.51	<u>25.5</u>	<u>37.3</u>	<u>37.3</u>	<u>55.9</u>	<u>37.3</u>
<u>15</u>	0	60	40	0.19	0.67	0.50	<u>13.9</u>	<u>49.2</u>	<u>36.9</u>	<u>67.7</u>	<u>36.9</u>
16	40	0	60	0.50	0.19	0.67	36.9	13.9	49.2	38.5	49.2
17	20	20	60	0.35	0.35	0.67	25.5	25.5	49.1	50.0	49.1
<u>18</u>	0	40	60	0.19	0.50	0.67	<u>13.9</u>	<u>36.9</u>	<u>49.2</u>	<u>61.5</u>	<u>49.2</u>
19	20	0	80	0.33	0.19	0.83	24.6	13.9	61.5	44.6	61.5
20	0	20	80	0.19	0.33	0.83	13.9	24.6	61.5	55.4	61.5
21	0	0	100	0.17	0.17	1.00	12.5	12.5	75.0	50.0	75.0

dark, No.13: dull², No.14: dull¹, No.15: pale, No.16: deep, No.17: hue (red), No.18: light, No.19: vivid², No.20: vivid¹, and No.21: very vivid. These numbers correspond to the suffix of fuzzy input I.

Table 2 shows the fuzzy inference results for 21 fuzzy inputs. The fuzzy inputs on the NCS are constructed by the centers (x_1' , x_2' , x_3') in the same manner as the crisp inputs in Table 1 and the radius ($r = 23\%$) of the basal plane (circle) of the cone indicated vagueness. (See Fig.3.) The inputs and outputs are not proportional, which indicates the nonlinear information processing in this fuzzy system.

Figure 5 illustrates a relationship between the lightness z_1 and the saturation z_2 obtained from data (z_1' , z_2') in Table 1 and 2. Squares (FC) indicate outputs for fuzzy inputs of chromatic colors (including reddish achromatic colors), circles (CC) indicate outputs for crisp inputs of chromatic and achromatic colors corresponding to Fig.2c. The outputs (circle points) are the same coordinates for the input points in the NCS system. The outputs (squares points) for fuzzy inputs are grouped at the center of the tone triangle. The points are clearly different. Vague color inputs to the NCS system (Fig.2a), the system outputs crisp color in the NCS system (Fig.2b), and also outputs crisp color

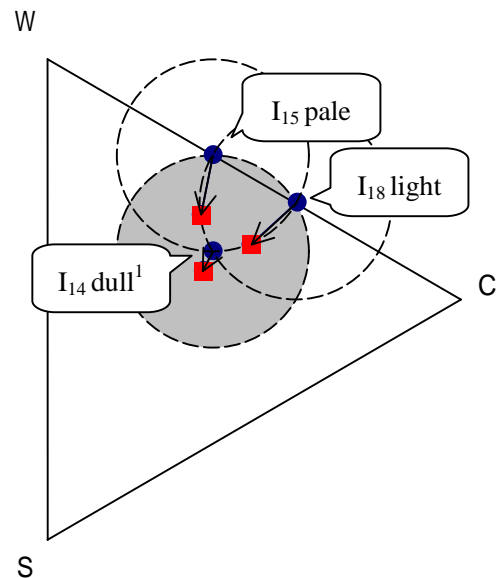


Fig.6. Relationship between lighter dull (dull¹), pale, and light on the NCS color triangle corresponding to Fig.2b

in the HLS system (Fig.2c). The coordinates of crisp inputs are the same as those of crisp outputs.

Figure 6 illustrates the differences between crisp and fuzzy inputs. Large circles show fuzzy inputs I_{14} , I_{15} , and I_{18} , which are JIS-like tone modifiers: “dull¹” (lighter dull), “pale”, and “light”, respectively. (See squared numbers and underlined values in Table 2.) Three arrows show changing directions from outputs for crisp to those for fuzzy (large circle). For I_{14} : dull¹, the arrow indicates a change toward blackness S. Output (square) for fuzzy input is dislocated from the center of fuzzy input (shaded large circle). On the other hand, output for crisp input is located at the center. Crisp input and its inference output are in the same coordinate. For “pale” and “light”, each output for fuzzy inputs is distant from the centers of large circles and is quite close to the “dull¹” centers, rather than the other centers. This implies that “pale” and “light” change to “dull¹”, if such vague colors are input into the fuzzy system.

When an inference output falls within the shaded circular area, closeness can be obtained simply from the distance between the center of the circle (crisp input) and the inference output. Determining closeness is equivalent to calculating the ratio $(r - d) / r$, where r is the radius of the circle, and d is the distance from the fuzzy input as the center of the shaded large circle to the inference output. For I_{14} : $d_1 = 4.9$; $\beta_1 = 78.7\%$, for I_{15} : $d_2 = 8.3$; $\beta_2 = 63.9\%$, and for I_{18} : $d_3 = 9.3$; $\beta_3 = 59.6\%$. That is, the vagueness of dull¹ shows a closeness of 78.7% in dull¹, the vagueness of pale shows a closeness of 63.9% in dull¹, and vagueness of light shows a closeness of 59.6% in dull¹. This means that a closeness of more than 50% indicates the modifier dull¹ in this case. Therefore, dull¹ with vagueness can be referred to as widely dull, pale and light with vagueness can also be referred to as dull.

The relationship between crisp inputs (centers of large circles), conical fuzzy inputs (large circles), top areas of 0.5 level-sets of each modifier (medium circles), and fuzzy inference outputs (circles and squares) on the NCS color triangle is examined.

Figure 7 shows a schematic illustration, which focuses on one dull¹ (lighter dull) area. The centers of modifiers “pale” and “light” as the coordinates of crisp inputs move to the area (middle circle) of “dull¹”, because the vagueness of two modifiers is inferred. The vagueness (large circle) of dull¹ is also inferred, and then center point moves to the blackness direction.

4. Conclusions

The present paper proposes a fuzzy system that, for a known hue name (e.g., red), can extract crisp outputs of the NCS system (which is available for use in fuzzy set theory) and HLS (which is easy to show via graphs). It is difficult to construct such a fuzzy system on the tone plane in the HLS system, because the membership function of the lightness is quite complicated on the tone triangle. The system also extracts, in a simple

manner, the membership grades from the projection of a conical membership function of a vague color input. Three parameters associated with respective grades indicate vague colors and output the center of gravity as a crisp color value although the NCS system does not have a ‘lightness’ attribute.

References

- [1] Japanese Industrial Standards Committee: *Names of non-luminous object colours JIS Z8102*, Japanese Standard Association, 1985, in Japanese.
- [2] Japanese Industrial Standards Committee: *Colours specification: Specification according to their three attributes JIS Z 8721*, Japanese Standard Association, 1993, in Japanese.
- [3] Sivik, L.: *Color systems for cognitive research. In Color categories in thought and language*, C. L. Hardin and Luisa Maffi, Ed., Cambridge University Press, New York, pp.163-193, 1997.
- [4] Fairchild, M. D.: *Color appearance models*, Reading, Massachusetts, Addison Wesley Longman, Inc., 1998.
- [5] Sugano, N.: Color-naming system using fuzzy set theoretical method. *Proc. of the Fourth Asian Fuzzy Systems Symposium*, vol.1, pp.42-47, 2000.
- [6] Sugano, N.: Fuzzy color-naming system using conical membership function of tone modifier, *Biomedical Soft Computing and Human Sciences*, vol. 7, No. 1, pp. 47-52, 2001.
- [7] Sugano, N.: Color-naming system using fuzzy set theoretical approach. *Proc. of the 10th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE2001)*, pp. 13-16, 2001.
- [8] Sugano, N.: Evaluation and improvement of fuzzy color-naming system using neighboring JIS modifiers, *Biomedical Soft Computing and Human Sciences*, vol. 8, No. 1, pp. 21-27, 2002.

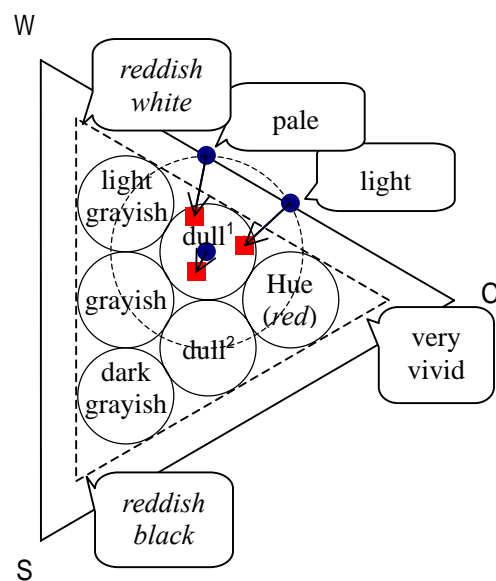


Fig.7. Schematic gathering effects of outputs for conical fuzzy input