Fuzzy Set Theoretical Analysis of Semantic Data as Human Membership Values on the Color Triangle

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***Abstract*—The present study considers a fuzzy color system in which three membership functions are constructed on the color triangle. This system can process a fuzzy input (as the membership values of subjects) to an RGB system and output the center of gravity of three weights associated with respective grades. Three input fuzzy sets (*red*, *green*, and *blue*) are applied to the color triangle relationship. By treating three attributes of redness, greenness, and blueness on the color triangle, an average color value can be easily obtained as the center of gravity of the fuzzy output. The subjects are required to determine whether the impressive word (or the color word) for each color is suitable. The response of subjects can be obtained as fuzzy sets (as the membership values of subjects) on the color triangle. The responses for impressive words (stimulus, appetite, concentration, relaxation, calmness, rest, and mild) and the responses for color words (red, orange, yellow, green, blue, purple, and pink) show mountain-like shape. The height of fuzzy set indicates possibility of a target color or degree of cognition for a target color and the expanse of fuzzy set shows vagueness, the fuzzy sets for a impressive word (e.g. stimulus) and for a color word (e.g. red) are compared with calculating an intersection of those fuzzy sets. The fuzzy sets for impressive words show large vagueness and small possibility rather than those for color words. The differences between impressive fuzzy sets and color fuzzy sets are described, and the relationship between the centers of gravity of fuzzy inputs and inference outputs for fuzzy inputs are shown in the present paper.**

I. INTRODUCTION

A technique for obtaining expressions of the color triangle using the fuzzy set theoretical method has been reported [2] and improved [3]. In the previous study, the relationship between input fuzzy sets with a plateau on the color triangle and fuzzy inputs of conical membership functions was examined. The color triangle represents the hue and saturation of a color [5]. The six fundamental colors and white can be represented on the same color triangle (See Fig. 1). Vague colors on the color triangle were clarified. In the present study, the membership value on the RGB triangular system are examined to determine the average color value as the center of gravity of the attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color identification, and similar applications.

II. COLOR TRIANGLE

Additive color mixing occurs when two or three beams of differently colored light combine. It has been found that mixing just three additive primary colors, red, green, and blue, can produce the majority of colors. In general, a color vector can be described by certain quantities as a scalar and a direction. These quantities are referred to as the tri-stimulus values, R for the red component, G for the green component, and B for the blue component, and are given as follows:

$$\vec{C} = \vec{R} + \vec{G} + \vec{B} \tag{1}$$

This is called the RGB color model. This concept allows colors to be represented by a planar diagram. The first step is to draw the red, green and blue components (R, G, B) as the vertices of a color triangle, as in Fig. 1. The coordinates on the plane of the color triangle can specify various colors. The location given by the coordinates corresponds to the amounts of r, g and b that make up the color. The coordinates specifying the center of the color triangle represent the case in which the three primary colors are mixed in equal proportion and indicate the color white. Such representations are called chromaticity diagrams. The diagram represents hue and saturation but not lightness [6]. On the color triangle, the percentages of redness, greenness, and blueness, where the total of the three attributes is equivalent to 100%, specify a color. In order to indicate only the direction of a color vector, i.e., the chromaticity, the redness r, greenness g, and blueness b are obtained as follows:

$$r = \frac{R}{R+G+B} \tag{2}$$

$$g = \frac{G}{R+G+B} \tag{3}$$

$$b = \frac{B}{R+G+B} \tag{4}$$

$$r + g + b = 1 \tag{5}$$

In other words, the direction is shown as the ratio of tri-stimulus values R, G, and B. The total of these ratios is equal to 1, as shown in Eq. (5).

III. METHODS

A. *Color triangle designs*

The previous study [2], [3], considered a system of the three primary colors, red, green, and blue (RGB), presented on an color triangle. As Fig. 1 shows, blue, cyan, green, yellow, red, magenta, and white are abbreviated as B, C, G, Y, R, M, and W, respectively. Six fundamental color coordinates, e.g., (r_1, g_1, b_1), (r_6, g_6, b_6), (r_{11}, g_{11}, b_{11}), …, were selected, where r_n, g_n, and b_n are the red, green, and blue attributes, respectively, of the n^{th} color.

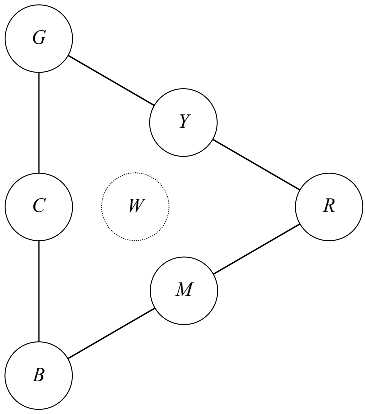

Fig. 1. A color triangle: A point on the plane of the triangular system represents the hue and saturation of a color spaces.

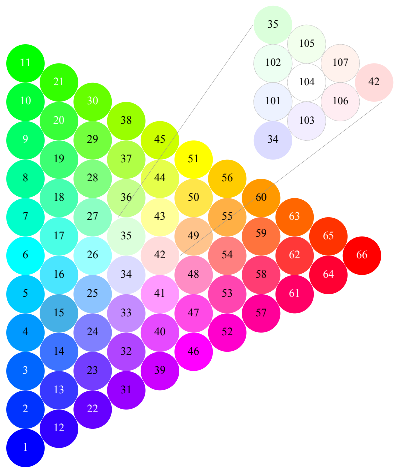

Fig. 2. Sixty-six crisp color inputs (*fundamental type*) and white with six neighboring colors (*detail type*) on the color triangle.

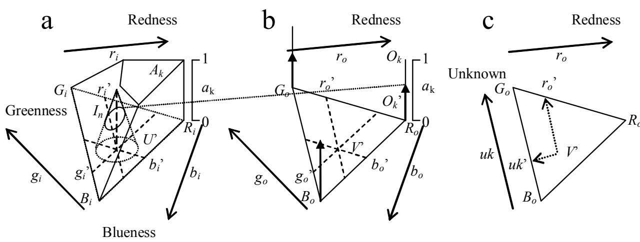

Fig. 3. Fuzzy system using the membership function of input fuzzy sets A_k, output crisp sets O_k and conical fuzzy input I_n on the color triangle.

Figure 2 corresponds to the schematic diagram shown in Fig. 1. The color names in Fig. 2 are No .1: blue, No .6: cyan, No. 11: green, No. 46: magenta, No. 51: yellow, and No. 66: red.

White (No. 104) is surrounded by six neighboring colors, as shown in the detail inset, and these seven colors (No. 101–No. 107) are surrounded by No. 34, No. 35, and No. 42.

B. Fuzzy rules

Figure 3 illustrates input fuzzy set, fuzzy input on the color triangle (Fig. 3a) and output crisp set, fuzzy output on the RGB triangle (Fig. 3b), and crisp output on the graphic plane (Fig. 3b). The fuzzy rules are as follows (See Figs. 3 and 6):

$$R^1 : if\ U\ is\ A_1\ then\ V\ is\ O_1 \tag{6}$$

$$R^2 : if\ U\ is\ A_2\ then\ V\ is\ O_2 \tag{7}$$

$$R^3 : if\ U\ is\ A_3\ then\ V\ is\ O_3 \tag{8}$$

Rule R^k : *if U is* A_k*, then V is* O_k ($k = 1, 2, 3$), where k is the rule number corresponding to the components of r, g, and b, A_k is a fuzzy set of inputs, O_k is a crisp set of outputs, $U = (r_i, g_i, b_i)$ are input coordinates (variable), and $V = (r_o, g_o, b_o)$ are output coordinates. Here, U and V are fixed to the color triangle and the RGB triangle. A fuzzy set A_k of inputs shows a triangular pyramid-like shape with a plateau at corner points R_i, G_i, and B_i, and a crisp set O_k of outputs of rule R^k is shown at corner points R_o, G_o, or B_o (a fuzzy set O_k' indicated by vertical arrows in Fig. 3*b*) on the color triangle, and the output is O_k if the input is A_k.

The fuzzy inference method is as follows. Let the inputs be $r_i = r_i$', $g_i = g_i$', and $b_i = b_i$'.

1) The input of rule R^k, grade $\alpha_k = A_k(U')$, where $k = 1, 2, 3$.

2) The output of rule R^k, output crisp set is shown as a vertical post.

3) $O_k' = \alpha_k O_k$, where O_k' is fuzzy sets (as vertical arrows) and O_k is crisp sets (as vertical posts) in Fig. 3*b*. The complete inference results O' of rules R^1, R^2, and R^3.

$$O' = \alpha_1 O_1 \cup \alpha_2 O_2 \cup \alpha_3 O_3 = O_1' \cup O_2' \cup O_3' \tag{9}$$

The output coordinate, $V' = (r_o', g_o', b_o')$ is equivalent to the center of gravity of the output fuzzy sets of O'. In addition, in Fig. 3*c*, $V' = (r_o', uk')$ corresponds to a coordinates of the graphic system, where uk' (on the unknown axis) is calculated from g_o' and b_o'. uk' shows a value (as distance from B) on the line B-G.

An input fuzzy set A_1 of *redness* can be characterized by the following membership function:

$$\mu_1(r_i, uk) = r_i s; \quad r_i < \frac{1}{s} \tag{10}$$

$$\mu_1(r_i, uk) = 1; \quad r_i \geq \frac{1}{s} \tag{11}$$

where s is slope of projection and s ranges from 0.02 to 0.03.

C. Fuzzy sets

The membership value $\mu_k(r_i', g_i', b_i')$ of input fuzzy set A_k on the color triangle is based on the values of seven colors (R, Y, G, C, B, M, and W). The membership value $\mu_k(r_i', g_i', b_i')$ is equal to $\mu_k(r_i', uk')$.

In Fig. 4, the shape of membership function is shown by including to W_i (white). Top of the plateau is shown as diamond-like shape in this case. See also Fig. 6.

Figure 5*a* (left) illustrates twenty-one fuzzy inputs (I_1–I_6, I_{12}–I_{16}, I_{22}–I_{25}, I_{31}–I_{33}, I_{39}–I_{40}, and I_{46}) on the color triangle as a triangle with color names (B, C, and M). The fuzzy inputs are formed by conical membership functions, and the fuzzy sets are made to mutually overlap. The edge of the basal plane (circle) of the conical fuzzy set passes through the centers of the overlapped circles.

Figure 5*b* (right) shows the arrangement of numbers corresponding to the conical fuzzy sets of Fig. 5*a*, and the numbers are shown inside circles representing the top of the 0.5 level-set (bottom-right). The color names are No. 1: blue, No. 6: cyan, and No. 46: magenta.

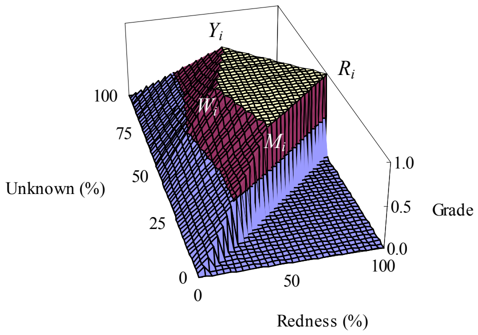

Fig. 4. The membership function μ_1 (r_i, uk) of input fuzzy set A_1 (*redness*) on the color triangle. This is corresponding to Fig. 3*a*.

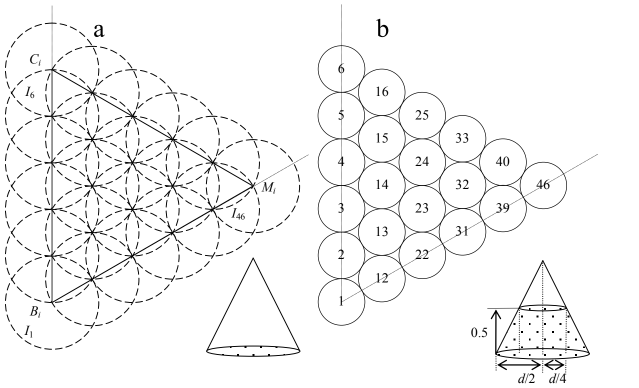

Fig. 5. Fuzzy inputs on part of the color triangle and top areas of 0.5 level-sets indicated by number. The diameter (d = 23.0%) of the basal plane (*circle*) of the cone indicated vagueness.

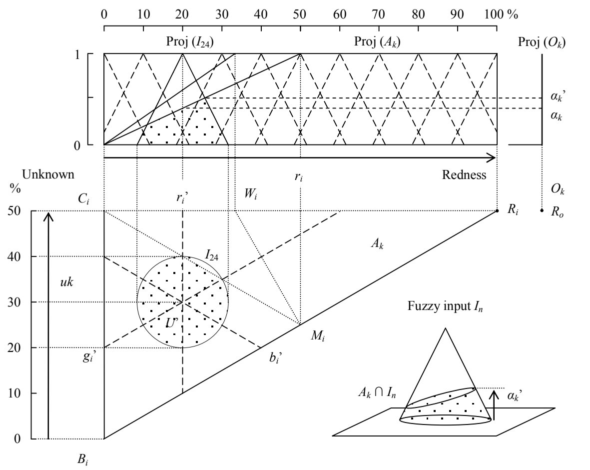

Fig. 6. Membership functions of input fuzzy sets A_k on half of the color triangle and one of sixty-six conical fuzzy inputs (*vague colors*).

TABLE I. NUMBER OF SUBJECTS IN THE EXPERIMENT.

Word type	No. of subjects	Male	Female	Age	Year
Impressive	68	34	34	4-60	2008
Impressive	59	22	37	5-65	2009
Color	86	47	39	6-70	2009

P method is used for impressive words and SD method is used for color words.

Figure 6 illustrates half of the color triangle as a base of input fuzzy set A_k and one of the sixty-six conical fuzzy inputs (I_1–I_{66}) on the color triangle. For k = 1 (as redness), sharp slant line (s = 0.03) shows a projection of line between C_i with membership value $\mu_k = 0$ and W_i with $\mu_k = 1$ and gentle slope line (s = 0.02) shows a projection of line between B_i with value $\mu_k = 0$ and M_i with value $\mu_k = 1$ (or between G_i with $\mu_k = 0$ and Y_i with $\mu_k = 1$ on the blind side). See also Fig. 4. The triangular membership function Proj(I_{24}) on the redness axis is one of eleven projections of the sixty-six fuzzy inputs (I_1–I_{66}) by the rays from the lower part, and the triangular membership function Proj(I_n) on the unknown axis is not used in this study.

The intersection of input fuzzy set A_k for fuzzy input I_n is $A_k \cap I_n$. (See the dotted area at the bottom-right of Fig. 6.) Grade α_k' = height ($A_k \cap I_n$). If the input is crisp, α_k' becomes α_k. R_o is the new red as output. Proj(O_k) is a projection of an output crisp set at the corner point R_o (See Fig. 3*b*).

The system considered in this study can translate input data U of vague color to output data V of simple color on the RGB triangle. The fuzzy input on the RGB is constructed by the center $U' = (r_i', g_i', b_i') = (0, 0, 100)$ in % and the diameter d of the basal plane (circle) of the cone indicated vagueness.

IV. EXPERIMENTAL METHODS

For the experiment, 213 (in Table I) undergraduate students, graduate students, and participants in a university festival volunteered to participate as subjects for this study. The subjects sat in a chair and were requested to watch a display continuously.

In this study, using a graphical user interface (GUI) for the questionnaire, 86 subjects compared the differences between a target color (e.g. red) and neighboring colors (e.g. red-relevant) of 65 colors (Fig. 2) using semantic differential (SD) method for color words and 127 subjects decided the boundary of target colors (e.g. stimulus) and neighboring colors (e.g. not stimulus) using partition (P) method for impressive words. That is, the subject partitions off 66 colors into two areas (e.g. stimulus or not stimulus).

Using the ensemble average of the fuzzy sets obtained from the stimulus-relevant colors, appetite-relevant colors, concentration-relevant colors, relaxation-relevant colors, calmness-relevant colors, rest-relevant colors, and mild-relevant colors, then, the normalized membership values of subjects are computed [4].

Using the ensemble average of the fuzzy sets obtained from the red-relevant colors, green-relevant colors, blue-relevant colors, yellow-relevant colors, cyan-relevant colors, magenta-relevant colors, and white-relevant colors, then, the normalized membership values of subjects are computed [4].

In practice, the subjects watch only 66 and 7 (in the detail inset) colors. The size of color of a circular shape is kept two degrees in sight. Seven groups of the colors for impressive words or for color words as the fuzzy sets are determined. The experiments were performed in an isolated area in order to restrict visual cues with regard to the display.

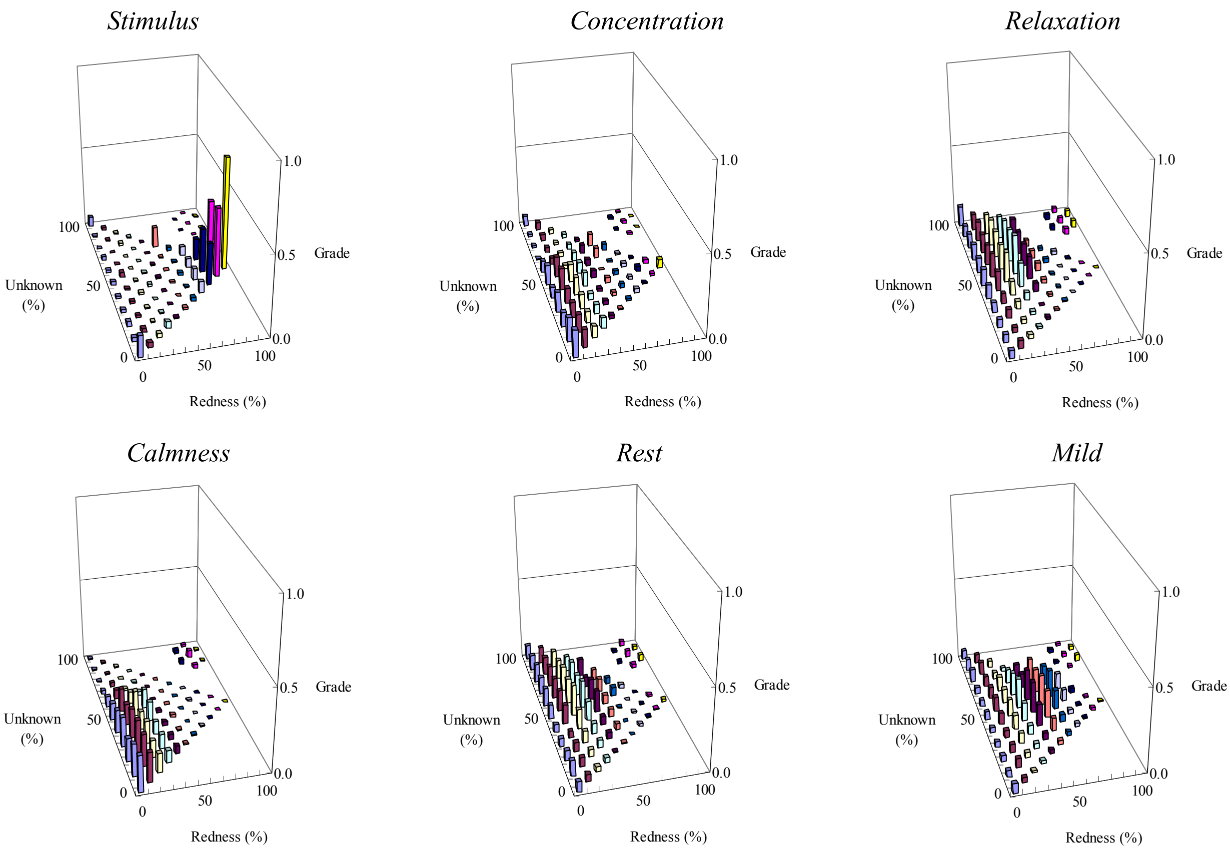

Fig. 7. Membership values of impressive words-relevant colors as fuzzy input.

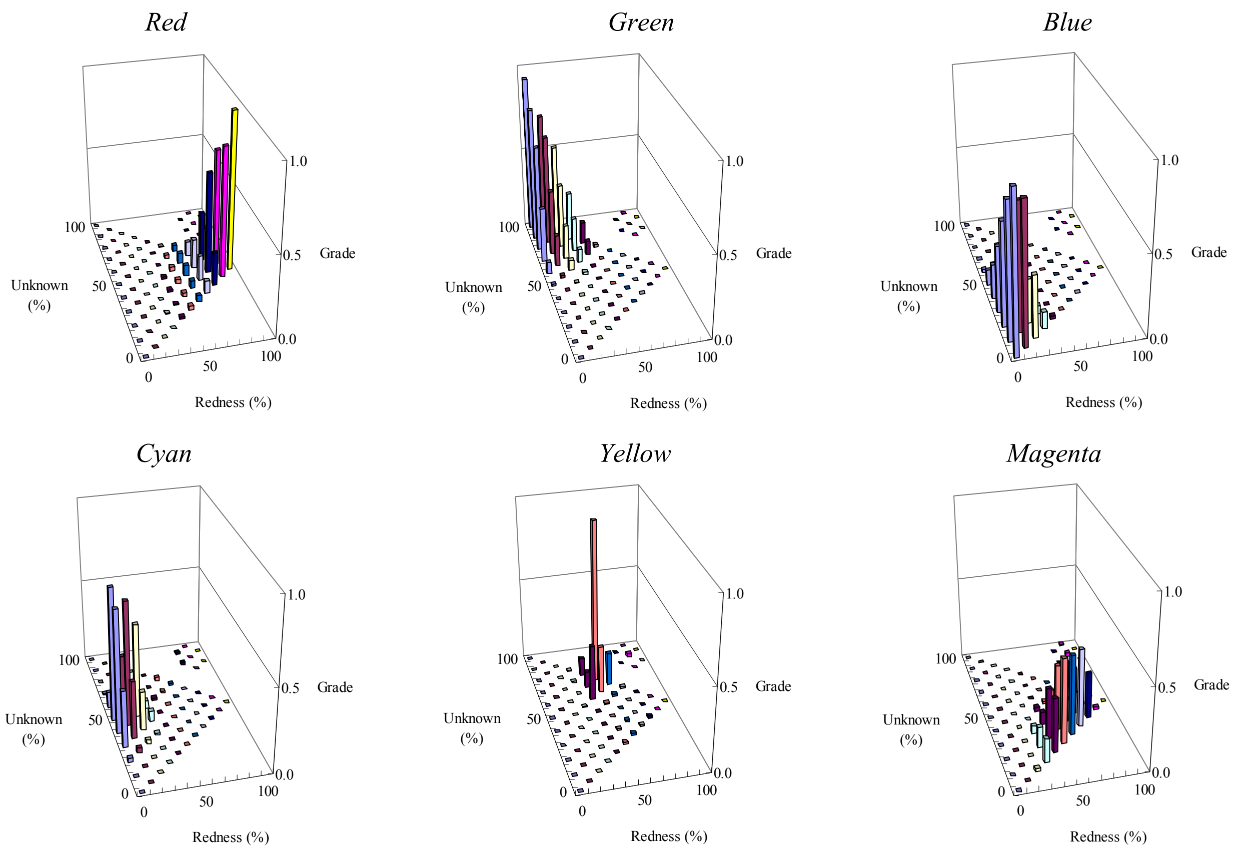

Fig. 8. Membership values of color words-relevant colors as fuzzy input.

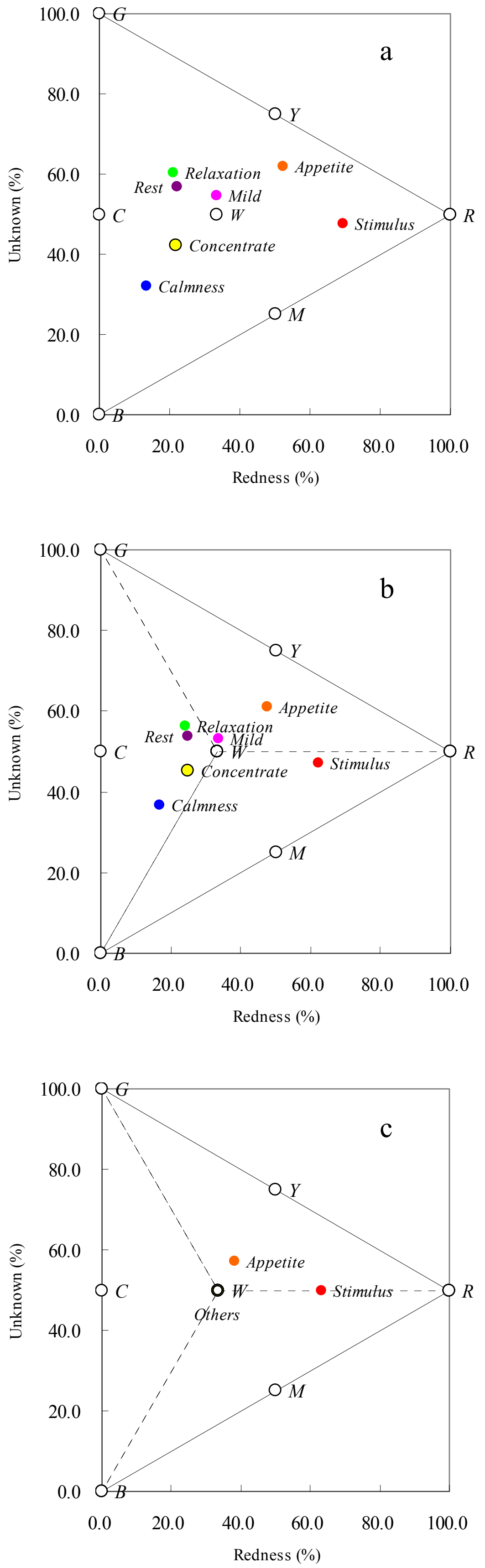

Fig. 9. *a*: Centers of gravity of impressive fuzzy sets (*filled circles*). *b*: Inference outputs (*filled circles*) for crisp inputs as the centers of gravity of fuzzy inputs, and *c*: inference outputs (*filled circles*) for fuzzy inputs.

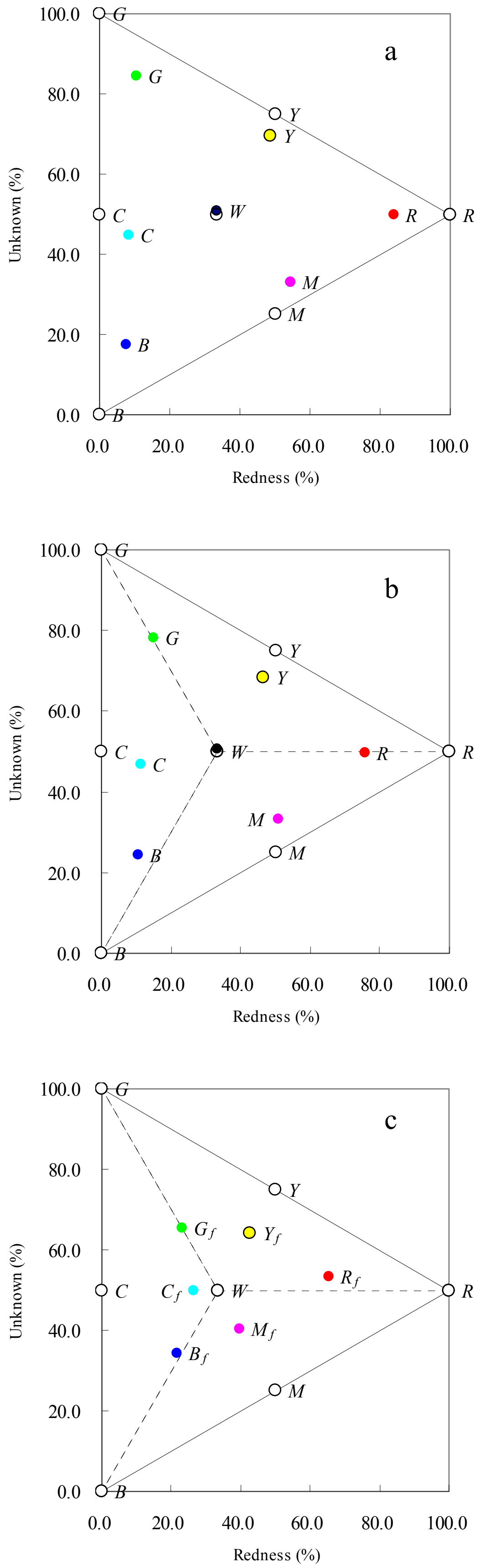

Fig. 10. *a*: Centers of gravity of color fuzzy sets (*filled circles*). *b*: Inference outputs (*filled circles*) for crisp inputs as the centers of gravity of fuzzy inputs, and *c*: inference outputs (*filled circles*) for fuzzy inputs.

TABLE II. INTERSECTIONS OF FUZZY SETS FOR IMPRESSIVE WORDS AND COLOR WORDS.

Impressive word	*Color word*						
	R	*G*	*B*	*C*	*Y*	*M*	*W*
Stimulus	0.69	0.06	0.13	0.03	0.13	0.26	-
Appetite	0.23	0.09	0.03	0.02	0.25	0.09	-
Concentrate	0.06	0.07	0.17	0.13	0.08	0.06	-
Relaxation	0.03	0.19	0.10	0.19	0.14	0.06	-
Calmness	0.02	0.06	0.22	0.18	0.18	0.09	-
Rest	0.06	0.15	0.10	0.19	0.19	0.06	-
Mild	0.09	0.09	0.08	0.10	0.10	0.16	-

V. EXPERIMENTAL RESULTS AND DISCUSSION

The experimental results for GUI are shown in this paper. Figure 7 shows the experimental results for stimulus-relevant colors, concentration-relevant colors, relaxation-relevant colors, calmness-relevant colors, rest-relevant colors, and mild-relevant colors, excluding to appetite-relevant colors, on the coordinates (r_i, uk) in fundamental type (66 colors) using P method. The membership values of 127 subjects are combined. The fuzzy sets of stimulus-relevant colors have a peak (Fig. 7). Others have not a shape peak and show large vagueness.

Figure 8 shows the experimental results for red-relevant colors, green-relevant colors, blue-relevant colors, cyan-relevant colors, magenta-relevant colors, and yellow-relevant colors, excluding to white-relevant colors, on the coordinates (r_i, uk) in fundamental type (66 colors) using SD method. The membership values of 86 subjects are combined. The fuzzy sets of red-relevant colors, green-relevant colors, and blue-relevant colors have a peak in Fig. 8. Yellow-relevant colors are gathered round coordinate (50, 75), and white-relevant colors in detail type have a sharp peak. However, cyan-relevant and magenta-relevant colors have a plateau peak (not sharp peak) at coordinate (0, 50) and (50, 25).

The calculation of intersections between membership values (divided into red, green, blue, yellow, cyan, and magenta) and the membership function of input fuzzy set are performed. See Eqs.6–9. A fuzzy set of red-relevant colors for instance is a subset of input fuzzy set of redness in this case.

Table II shows intersections between fuzzy sets for impressive words and color words calculated from Figs. 7 and 8. The intersection of the word “Stimulus” to the word “Red” indicates high degree (70%).

Figures 9 and 10 illustrate a relationship between the redness value r_i and the unknown value *uk*. *a*: Filled circles indicate the centers of gravity for fuzzy sets. A center (average) of fuzzy input is shown as a trend. Open circles indicate crisp sets of colors (as target colors). Target color in Fig. 9 means the corresponding impressive color (stimulus-red, appetite-orange, concentration-yellow, relaxation-green, calmness-blue, rest-purple, and mild-pink) [1] indicated colored circles. Target color in Fig. 10 means the fundamental colors indicated open circles. The differences between target colors (open circles) and the centers of gravity (filled circles) are large in Fig. 9 and not so large in Fig. 10. The relaxation, rest, and mild are gathered in Fig. 9.

In Figs. 9*b* and 10*b*, filled circles indicate the inference outputs for crisp inputs as the centers of gravity of fuzzy inputs. Open circles also indicate crisp inputs of colors (as target colors). In *c*, filled circles indicate the inference outputs for fuzzy inputs, open circles also indicate crisp inputs of colors (as target colors). The outputs (filled circles) for fuzzy inputs are grouped at the center of the color triangle. The open and filled circles are clearly different in this case. The differences between target colors (open circles) and the outputs (filled circles) are so large. The differences in *b* are smaller than those in *c*. The gathering effects exist [2]-[5].

Vague color inputs (Figs. 7 and 8) to the fuzzy system, the system outputs crisp color in the RGB triangle, and also outputs crisp color on the graphic plane (Figs. 9 and 10). These inference results for fuzzy 496 colors in previous study [5] and fuzzy 66 colors in this study are similar (Fig. 10). These inference results for partition method [4] and semantic differential method [5] are also similar (Fig. 10).

VI. CONCLUSION

The present paper examined how vagueness is presented on the color triangle and performed fuzzy set theoretical analysis. The subjects were asked the boundary of fuzzy sets for impressive words (e.g. stimulus) using partition method and also asked the boundary of fuzzy sets for color words (e.g. red) using semantic differential method. Performing the fuzzy inference for semantic data, it is found that these results move to white direction as a center of color triangle. The fuzzy sets for impressive words show large vagueness and small possibility rather than those for color words in the present study. Stocking such a data the human-computer interaction will be going well with the environment.

REFERENCES

[1] S. Noda, *Iro de kufuusuru herusyi raifu*, Rakuraku kenkoujyutsu, Mainichi shinbun, Tokyo, May 2002, in Japanese.

[2] N. Sugano, Fuzzy set theoretical approach to the RGB color triangle, in *Knowledge-Based Intelligent Information and Engineering Systems*. B. Gabrys, R. J. Howlett, and L. C. Jain, Eds. Lecture Notes in Computer Science, Springer-Verlag, Berlin Heidelberg, Part III, LNAI vol. 4253, October 2006, pp. 948-955.

[3] N. Sugano, Fuzzy set theoretical approach to the RGB triangular system, *Journal of Japan Society for Fuzzy Theory and Intelligent Informatics*, vol. 19, no. 1, pp. 31-40, February 2007.

[4] N. Sugano, Y. Chiba, Fuzzy set theoretical analysis of the membership values on the RGB color triangle, *Proc. of the IEEE International Conference on Systems, Man, and Cybernetics*. Montreal, pp. 841-846, October 2007.

[5] N. Sugano, S. Komatsuzaki, H. Ono, Y. Chiba, Fuzzy Set Theoretical Analysis of Human Membership Values on the Color Triangle, *Journal of Computers*, Academy Publisher, Vol.4, No.7, 593-600, July 2009.

[6] R. J. D. Tilley, *Colour and Optical Properties of Materials, An exploration of the relationship between light, the optical properties of materials and colour*. John Wiley & Sons, New York, 1999.