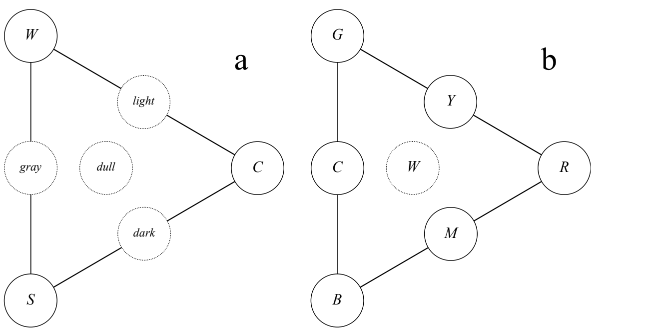


Fig. 1. A tone triangle (*a*). A point in the plane of the triangular system represents the lightness and saturation of a color. *S* is black. *C* is maximum chroma of each hue. A color triangle (*b*). A point in the plane of the triangular system represents the hue and saturation of a color. *C* is cyan.

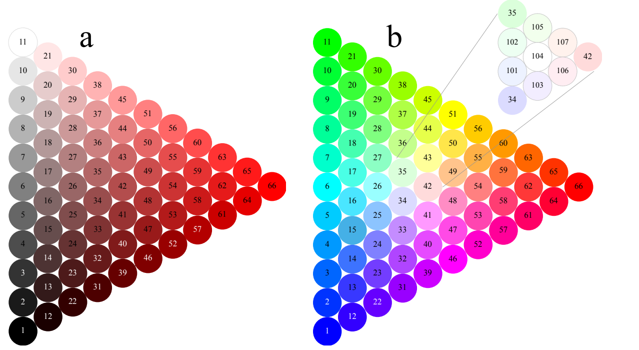


Fig. 2. Sixty-six crisp color inputs on the tone triangle (*a*). Sixty-six crisp color inputs and white with six neighboring colors (*detail*) on the RGB color triangle (*b*).

TABLE I
FUZZY RULES FOR SIX FUNDAMENTAL COLORS

Hue color	Input fuzzy set			Output crisp set		
	R^1	R^2	R^3	R^1	R^2	R^3
Red	A_1	A_2	A_2	O_1	O_2	O_3
Green	A_2	A_1	A_2	O_1	O_2	O_3
Blue	A_2	A_2	A_1	O_1	O_2	O_3
Yellow	A_1	A_1	A_2	O_1	O_2	O_3
Cyan	A_2	A_1	A_1	O_1	O_2	O_3
Magenta	A_1	A_2	A_1	O_1	O_2	O_3

Table I shows the fuzzy rules of six fundamental colors. Three rules are composed of two membership functions in this case.

The fuzzy inference method is as follows. Let the inputs be $c = c'$, $w = w'$, and $s = s'$.
1) The input of rule R^k, grade $\alpha_k = A_j(U')$, where k and j are shown in Table I.
2) The output of rule R^k, the α_k level-set is shown as a vertical allow.
3) $O_k' = \alpha_k O_k$, where O_k' is fuzzy sets and O_k is crisp sets in Fig. 3b. The complete inference results O' of rules R^1, R^2, and R^3.

$$O' = \alpha_1 O_1 \cup \alpha_2 O_2 \cup \alpha_3 O_3 = O_1' \cup O_2' \cup O_3' \qquad (2)$$

The output parameter, $V' = (r', g', b')$, corresponds to the coordinates of the central axis of the membership function of O'. In addition, in Fig. 3c, $V' = (r', uk')$ corresponds to a coordinates of the graphical system, where uk' (on the vertical axis) is calculated from g' and b'. uk_o' shows a value (as distance from B) on the line B-G.

Figure 4 shows the membership function of input fuzzy sets A_1 (*anti-blackness*), A_2 (*whiteness*), and A_3 (*lightness*). See also Table I, Fig. 3, and Fig. 6. See also Table III.

Table II shows the membership value $\mu_j(c', w', s')$ of input fuzzy set A_j on the tone triangle. In Fig. 4 $\mu_j(c', w', s')$ is equal to $\mu_j(c', uk_3')$. uk_3 is equal to the lightness. The membership function μ_j was based on the three additive primary colors in this study. μ_j is corresponding to A_j in Figs. 4 and 10. See Appendix.

Table III shows the fuzzy rules of two typical colors. Three rules are composed of two or three membership functions in this case. Orange is yellow red (not used in this study). Lime is yellow green.

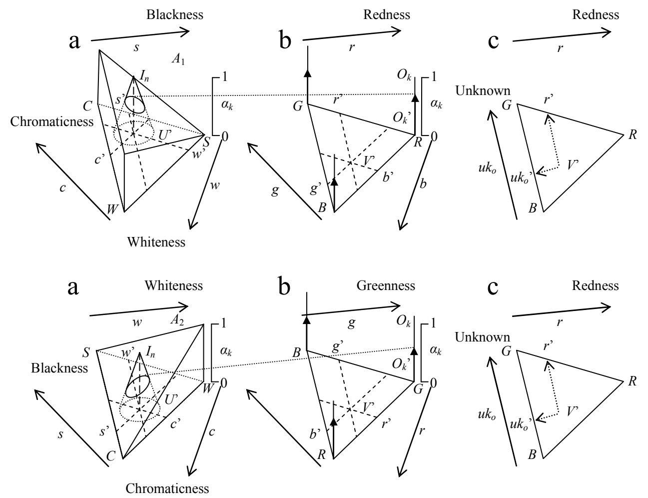


Fig. 3. Fuzzy system using the membership function of input fuzzy sets A_j, conical fuzzy input I_n on the tone triangle and output crisp sets O_k on the RGB color triangle. *a* (*upper trace*): input fuzzy set A_1 (*anti -blackness*) and *a* (*lower trace*): input fuzzy set A_2 (*whiteness*).

TABLE II
MEMBERSHIP VALUE OF INPUT FUZZY SET A_j ON THE TONE TRIANGLE

Color	Color coordinate			Membership value μ_j		
	c	w	s	j=1	j=2	j=3
C	1.00	0.00	0.00	1.00	0.00	0.50
W	0.00	1.00	0.00	1.00	1.00	1.00
S	0.00	0.00	1.00	0.00	0.00	0.00

C is maximum chroma of each hue. *S* is black.

TABLE III
FUZZY RULES FOR TWO TYPICAL COLORS

Hue color	Input fuzzy set			Output crisp set		
	R^1	R^2	R^3	R^1	R^2	R^3
Orange	A_1	A_3	A_2	O_1	O_2	O_3
Lime	A_3	A_1	A_2	O_1	O_2	O_3

A_3 is a shape between A_1 and A_2. Membership value of A_3 is 0.5 in the coordinate (1, 0, 0). See Table II.

Figure 5a illustrates twenty-one fuzzy inputs ($I_{46} - I_{66}$) on part of the tone color triangle with color names: dark (or deep), light (or pale), and C. The fuzzy inputs are formed by conical membership functions, and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapped circles. Fig. 5b shows the arrangement of numbers corresponding to the conical membership functions of Fig. 5a, and the numbers are shown inside circles representing the top of the 0.5 level-set (bottom-right). The color names or modifiers are No.46: dark (or deep), No.51: light (or pale) and No.66: C.

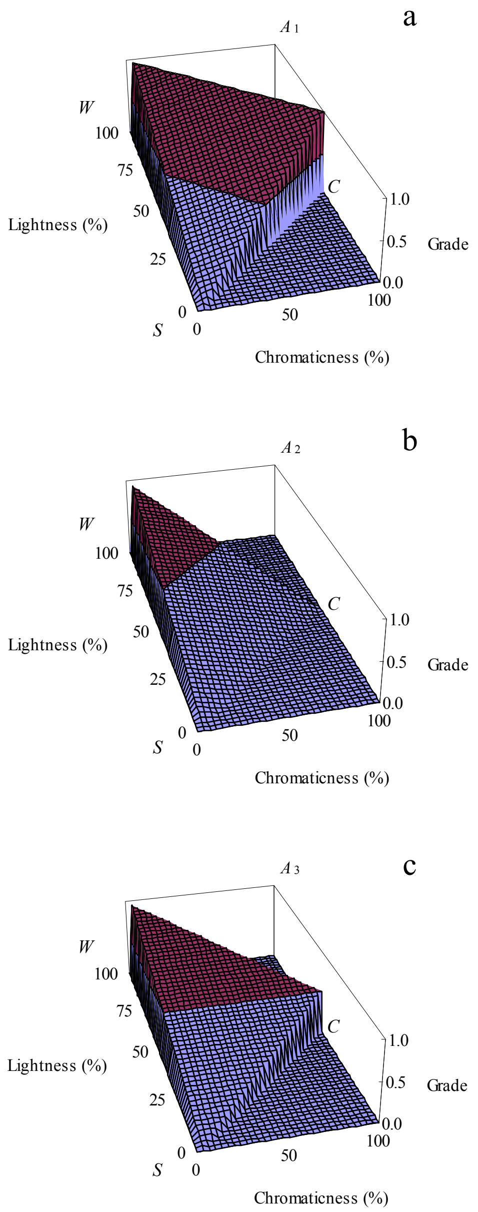


Fig. 4. Input fuzzy sets A_j and the membership functions μ_j. *a*: μ_1 (c, uk_3) of A_1 (*anti-blackness*), *b*: μ_2 (c, uk_3) of A_2 (*whiteness*), and *c*: μ_3 (c, uk_3) of A_3 (*lightness*) on the tone triangle. uk_3 is equal to the lightness.

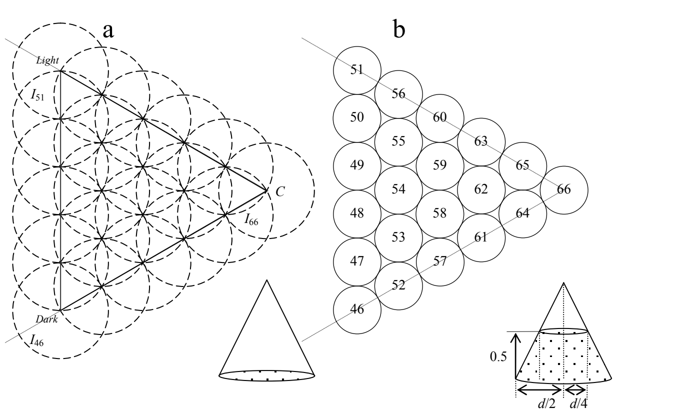


Fig. 5. Fuzzy inputs on part of the tone triangle (*a*) and top areas of 0.5 level-sets indicated by number (*b*). The diameter (d = 23.0%) of the basal plane (*circle*) of the cone indicated vagueness.

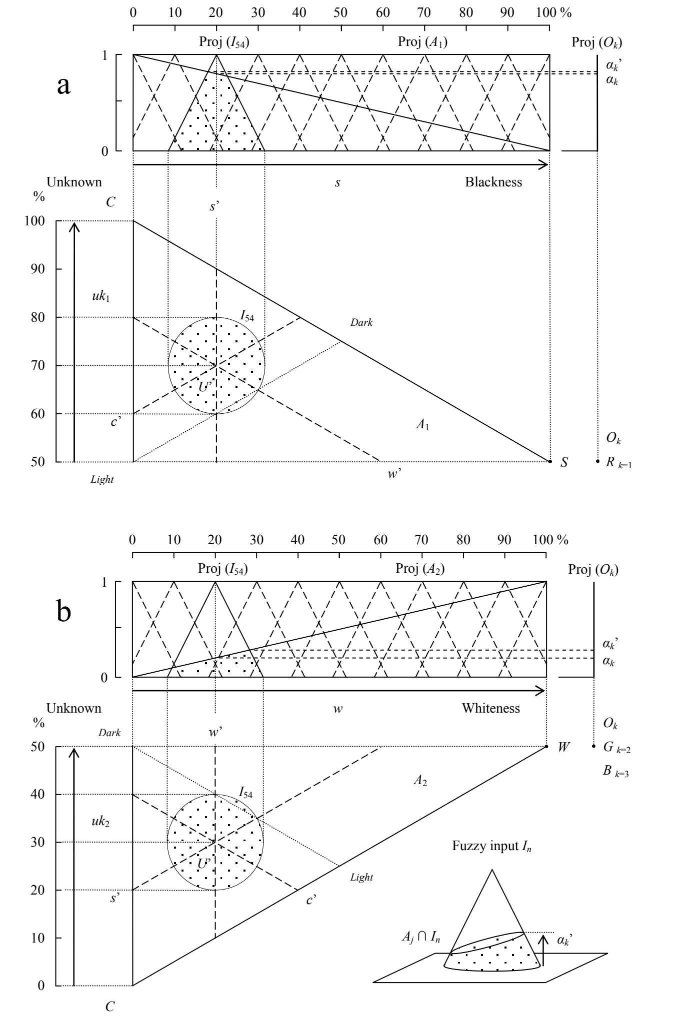


Fig. 6. One of sixty-six conical fuzzy inputs I_n (*vague colors*) and membership functions μ_j of input fuzzy sets A_j on half of the tone triangle. *a*: μ_1 (s, uk_1) = −0.01s + 1 on the projection of A_1. *b*: μ_2 (w, uk_2) = 0.01w on the projection of A_2.

Figure 6 illustrates half of the tone triangle as a base of input fuzzy set A_j and one of the sixty-six conical fuzzy inputs (I_1 – I_{66}) on the tone triangle. In the input fuzzy set A_1 (as *anti-blackness*), slope line shows a projection of line between S with membership value μ_1 = 0 and W with μ_1 = 1 (or between S with membership value μ_1 = 0 and C with μ_1 = 1 on the other side) and in the input fuzzy set A_2 (as *whiteness*) slope line shows a projection of line between W with value μ_2 = 1 and C

with value $\mu_2 = 0$ (or between W with value $\mu_2 = 1$ and S with $\mu_2 = 0$ on the other side). See also Table 1 and Fig. 4. The triangular membership function Proj (I_n) on the blackness axis (a) and Proj (I_n) on the whiteness axis (b) is one of eleven projections of the sixty-six fuzzy inputs ($I_1 - I_{66}$) by the rays from the lower part, and the triangular membership function Proj (I_n) on the unknown axis (uk_1 and uk_2) is not used in this study.

An input fuzzy set A_1 of *anti-blackness* can be characterized by the following membership function:

$$\mu_1 (s, uk_1) = -0.01s + 1 \quad (3)$$

where 0.01 is slope of projection. See Fig. 6a.

The limitations of uk_1 (on C-W side) are as follows:

$$50 \geq uk_1 \geq s/2 \quad (4)$$

$$50 < uk_1 \leq -(s/2) + 100 \quad (5)$$

An input fuzzy set A_2 of *whiteness* can be characterized by the following membership function:

$$\mu_2 (w, uk_2) = 0.01w \quad (6)$$

where 0.01 is slope of projection. See Fig. 6b. The limitations of uk_2 (on S-C side) are as follows:

$$50 \geq uk_2 \geq w/2 \quad (7)$$

$$50 < uk_2 \leq -(w/2) + 100 \quad (8)$$

An input fuzzy set A_3 of lightness can be characterized by the following membership function:

$$\mu_3 (c, uk_3) = 0.01uk_3 \quad (9)$$

where 0.01 is slope of projection on uk_3. The limitations of uk_3 (on W-S side) are as follows:

$$50 \geq uk_3 \geq c/2 \quad (10)$$

$$50 < uk_3 \leq -(c/2) + 100 \quad (11)$$

III. Experimental Methods

For the experiment, 166 (in Table IV) undergraduate students, graduate students, and participants in a university festival volunteered to participate as subjects for this study. The subjects sat in a chair and were requested to watch a display continuously.

First, the subjects picked up red-relevant colors, yellow -relevant colors, lime-relevant colors, green-relevant colors and blue-relevant colors from 496 colors in detail type. Five groups of the tone crisp sets are determined. Using the ensemble average as the tone fuzzy sets, then, the averaged membership values of 92 subjects are computed in present study [7].

Second, the subjects picked up red-relevant colors, yellow-relevant colors, lime-relevant colors, green-relevant colors and blue-relevant colors from 66 colors in fundamental type (See Fig.2a). Five groups of the tone crisp sets are determined in the same manner as 496 colors (detail type). The averaged membership values of 74 subjects are computed in present study [7].

TABLE IV
Number of Subjects in the Experiment

Type	No. of subjects	Male	Female	Age
496	92	32	60	10-70
66	74	26	48	4-14

Partition method is used. 496: detail type and 66: fundamental type.

Equipment

HP Compaq 14.1” Liquid Crystal Display and Panasonic 12.1” Liquid Crystal Display were used to present the stimulus pattern. The display resolution was 1024×768 pixels/60 Hz.

IV. Results and Discussion

The system considered in the present study can translate input data U of a vague color to output data V of a simple color on the RGB color triangle.

The intersection of input fuzzy set A_j for fuzzy input I_n is $A_j \cap I_n$. (See the dotted area at the bottom-right of Fig. 6b.) Grade α_k’ = height ($A_j \cap I_n$). If the input is crisp, α_k’ becomes α_k. R in Fig. 6a is the red, G or B in Fig. 6b is the green or the blue as output. Proj (O_k) is a projection of an output crisp set at the corner point R, G, or B (See Fig. 3b).

The fuzzy input (No.54) on the tone triangle is made up of the center U’ = (c’, w’, s’) = (60, 20, 20) in % and the diameter d = 23.0% of the basal plane (circle) of the cone indicated vagueness.

In previous study [2] the intersection of input fuzzy set A_j for fuzzy input I_n differed depending on whether or not I_n included the linear *edge* of A_j. The edges affected the nonlinear information processing. However, the edge effects are not considered in this study.

Figure 7 shows the experimental results for red-relevant colors, yellow-relevant colors, lime-relevant colors, green -relevant colors, and blue-relevant colors on the tone triangle in detail type. The membership values of 92 subjects are combined. Fundamental type (66 colors) is not displayed.

Figure 8 shows the centers of gravity of fuzzy inputs on the tone triangle. C is the maximum chroma of each hue. A center (average) of fuzzy input is shown as a trend. These inference results for fuzzy 496 colors (large filled circle) and fuzzy 66 colors (small filled circle) are similar.

It assumes that the fuzzy inputs on the RGB are constructed by the centers (r_i’, g_i’, b_i’). The center (R_i) of red-relevant colors is (100, 0, 0), the center (G_i) of green-relevant colors is (0, 100, 0), and the center (B_i) of blue-relevant colors is (0, 0, 100), , the center (Y_i) of yellow-relevant colors is (50, 50, 0),the center (L_i) of lime-relevant colors is (25, 75, 0), which assume to fuzzy input I_n in Fig.5a. The coordinates of centers and that of inference outputs are examined.

In Table V the crisp input means a singleton on the center of gravity of fuzzy input. In Tables V and VI, s is blackness, w is whiteness, l is lightness, and α_k is the grade of intersection for crisp input in Table V, α_k’ is the grade of intersection for fuzzy input in Table VI. See Fig. 6.

Figure 9 is obtained from Table V and VI. Fig. 9 illustrates the relationship between the vertical value uk_o and the redness value r obtained from data (r’, uk_o’). The circles indicate outputs for crisp inputs of colors in Fig. 9, corresponding to Fig. 3c. The inference outputs for crisp inputs are grouped at the center of the RGB color triangle. This effect is causing from shapes of membership function (triangular pyramid) and computing of the center of gravity (de-fuzzification). The results are the same as the results of previous study [2-6]. Namely the gathering effects for crisp inputs using input fuzzy sets of triangular pyramid are existed in previous study [2-6].

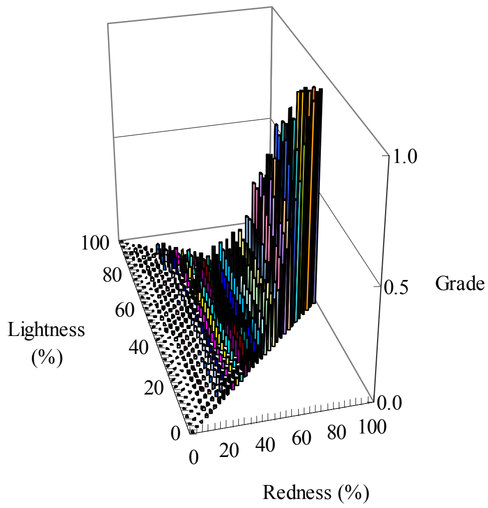


Fig. 7a. Membership values of red-relevant colors on the tone triangle.

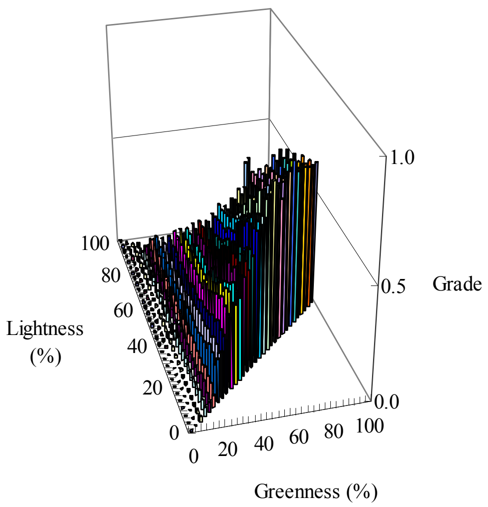


Fig. 7b. Membership values of green-relevant colors on the tone triangle.

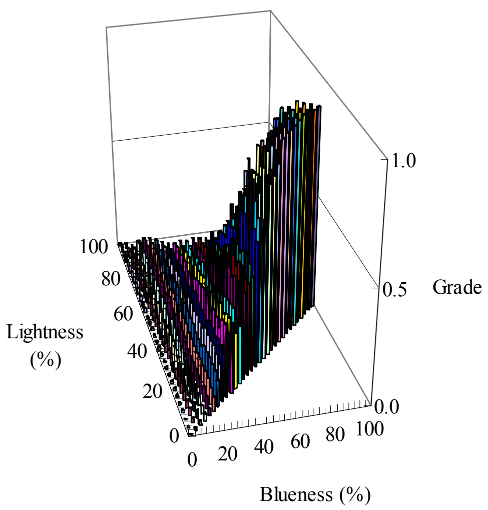


Fig. 7c. Membership values of blue-relevant colors on the tone triangle.

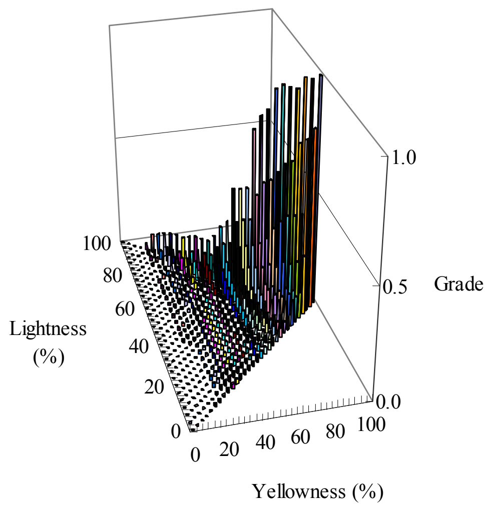


Fig. 7d. Membership values of yellow-relevant colors on the tone triangle.

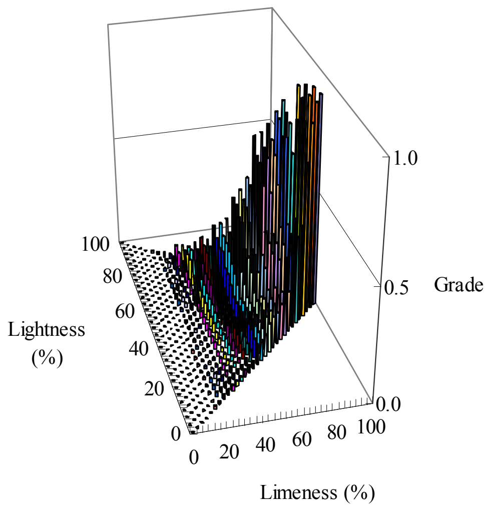


Fig. 7e. Membership values of lime-relevant colors on the tone triangle.

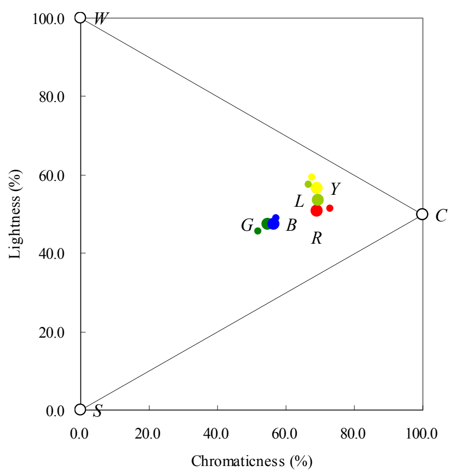


Fig. 8. The centers of gravity of fuzzy inputs on the tone triangle. *W* is white. *S* is black. *C* is maximum chroma of each hue (*open circle*). 496 type (*large filled circle*) and 66 type (*small filled circle*).

TABLE V

INTERSECTION OF CRISP INPUTS AND INPUT FUZZY SETS A_k

Color and type	Blackness, whiteness, lightness, and grade					
	A_1		A_2		A_3	
	s'	α_1	w'	α_2	l'	α_3
R496	14.70	0.85	16.13	0.16	-	-
G496	25.11	0.75	20.02	0.20	-	-
B496	24.17	0.76	19.26	0.19	-	-
Y496	8.97	0.91	21.95	0.22	-	-
L496	11.79	0.88	18.77	0.19	53.55	0.54
R66	12.17	0.88	15.02	0.15	-	-
G66	28.78	0.71	19.56	0.20	-	-
B66	22.42	0.78	20.24	0.20	-	-
Y66	6.74	0.93	25.67	0.26	-	-
L66	9.18	0.91	24.31	0.24	57.87	0.58

s is blackness, w is whiteness, and l is lightness in %, α_k is the grade of intersection for crisp input. 496 is detail type. 66 is fundamental type.

TABLE VI

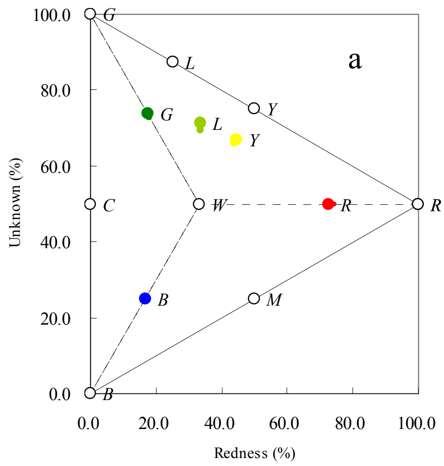
INTERSECTION OF FUZZY INPUTS AND INPUT FUZZY SETS A_k

Color and type	Blackness, whiteness, lightness, and grade					
	A_1		A_2		A_3	
	s'	α_1'	w'	α_2'	l'	α_3'
R496	6.09	0.94	32.65	0.33	-	-
G496	31.98	0.68	38.42	0.38	-	-
B496	11.00	0.89	37.50	0.38	-	-
Y496	0.00	1.00	36.88	0.37	-	-
L496	0.00	0.95	38.86	0.39	56.67	0.57
R66	0.00	0.93	36.44	0.36	-	-
G66	28.89	0.71	36.52	0.37	-	-
B66	15.00	0.85	39.59	0.40	-	-
Y66	0.00	1.00	23.23	0.23	-	-
L66	0.00	0.95	42.14	0.42	65.00	0.65

s is blackness, w is whiteness, and l is lightness in %, α_k' is the grade of intersection for fuzzy input. 496 is detail type. 66 is fundamental type.

V. CONCLUSIONS

The present paper examined how vagueness is presented on the RGB tone triangle using partition methods and performed fuzzy set theoretical analysis. The subjects are asked where partitions of fundamental colors are on the triangle. The subjects divide RGB tone triangle in two areas with a partition. The number of neighboring colors is different for each subject. It considered that the data of each subject is a crisp set, the ensemble average of all the data for each hue is a fuzzy set. The results of experiments show a similar trend for the RGB color triangle. Using the fuzzy inference for RGB tone data (as a fuzzy set), it is found that these results move to white direction as a center of RGB color triangle. Especially the inference results of the yellow-relevant colors and the lime-relevant colors show a different trend in the present study.



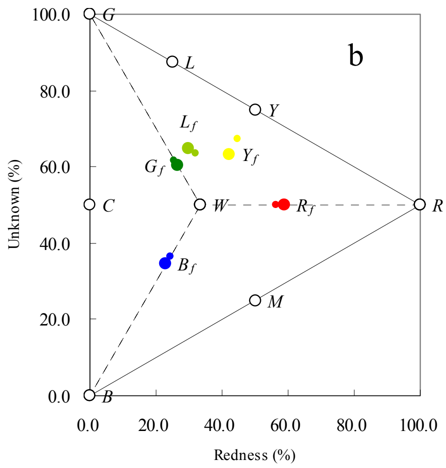


Fig. 9. Inference outputs (*open circle*) for crisp inputs, inference outputs for crisp inputs as the centers of gravity of fuzzy inputs (*a*), and inference outputs for fuzzy inputs (*b*) on the RGB color triangle. 496 type (*large filled circle*) and 66 type (*small filled circle*). Suffix *f* shows fuzzy inference output. White exists in the coordinates (33.3%, 50.0%).

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