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FUZZY SET THEORETICAL APPROACH TO ACHROMATIC RELEVANT COLOR ON THE NATURAL COLOR SYSTEM

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Abstract. *The present study considers a fuzzy natural color system (NCS) in which triangular pyramid membership functions are constructed on the NCS color triangle. This system can process a fuzzy input to an NCS and output a center of gravity of three weights associated with respective grades. Triangular membership functions are applied to the hue angle, and triangular pyramid membership functions are applied to the NCS color triangle relationship. By treating three membership functions of blackness, whiteness, and chromaticness on the NCS color triangle, a target color can be easily obtained as the center of gravity of the output fuzzy set. In the present paper, the differences among crisp achromatic colors, fuzzy achromatic colors, and fuzzy modified achromatic colors (e.g., reddish achromatic colors) are described, and crisp modified achromatic colors are shown to be a result of achromatic colors that have appropriate vagueness.*

Keywords: Fuzzy set theoretical approach, Natural color system (NCS), Achromatic color, Vague color, NCS color triangle, Triangular pyramid membership function, Conical membership function, HLS (hue, lightness, saturation) System

1. **Introduction.** Sivik introduced the Swedish Natural Color System (NCS), which is based on the principles of opponent-process theory [4]. The Munsell (HVC) system, widely used in the US and Japan, is based on lightness (Value), saturation (Chroma), and Hue. The goal in designing this system is to achieve equal perceptual distance within each attribute. Munsell divided the hue circuit into five sectors of twenty steps each, with the resulting principal hue divisions being purple, red, yellow, green, and blue. Estimating the degree of resemblance that a color sample bears to ideal red, yellow, green, blue, black, and white scales the NCS. Estimating its chromaticness, whiteness, and blackness as a percentage, where the total of the three attributes is equivalent to 100%, specifies a sample. There are several important differences between the Munsell system and NCS. NCS is based on direct estimation rather than on comparison with samples, so use of the NCS (rather than an NCS atlas) is independent of lighting conditions. Munsell is open with respect to the saturation dimension, whereas NCS is closed, i.e. NCS chromaticness can be estimated and Munsell Chroma cannot be estimated. Munsell space is based on differences between neighboring colors, NCS is based on the degree of resemblance to elementary colors. With regard to lightness, the Munsell system considers lightness to be fundamental, whereas, in contrast, NCS considers blackness to be fundamental. Two systems represent the psychological space by which people categorize colors.

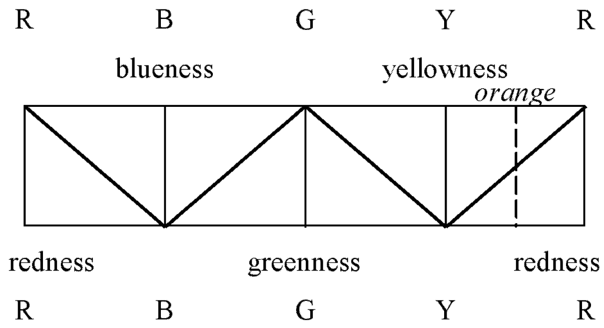

Figure 1. Hue dimension of four bi-polar scales

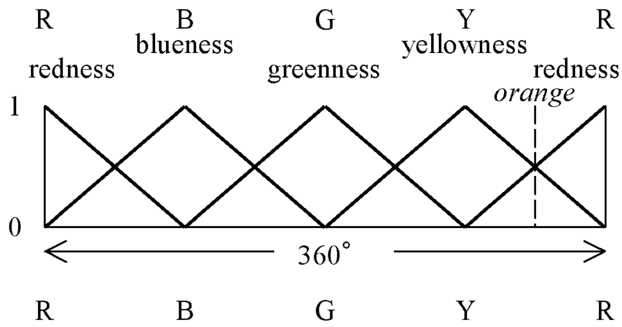

Figure 2. Triangular membership function of four hues

In the NCS, a method similar to the fuzzy set theoretical method for obtaining hue expressions with vagueness has been reported by Sivik [4] (see Figure 1). Using the fuzzy set theoretical method, a technique for acquiring tone expressions with vagueness on the NCS color triangle has been investigated by Sugano [9-11]. In a previous study, the triangular membership functions of achromatic colors and conical membership functions of chromatic colors were used as vagueness [9-11], which caused the gathering effect toward the center on the NCS tone. However, the relationship between achromatic colors, modified achromatic colors (e.g., reddish achromatic colors), and chromatic colors has not yet been clarified in detail [9, 11], and we are left with the following questions. What are modified achromatic colors? Do achromatic colors having vagueness produce modified achromatic colors in positions between achromatic colors and chromatic colors?

In the present study, fuzzy achromatic colors of triangular membership functions and fuzzy achromatic colors of conical membership functions are used on the NCS color triangle corresponding to the HLS tone plane consisting of lightness and saturation [5-8]. The vagueness effects of achromatic colors and modified (e.g., reddish, yellowish, greenish, bluish, etc.) achromatic colors are clarified [10]. Such a system will help us to determine the average color value as the center of gravity of the attribute information of vague colors. This fuzzy set theoretical approach is useful for vague color information processing, color-naming systems, and similar applications.

2. Natural Color System.

2.1. Hue circle and hue partition. The NCS adopts the traditional form of a circle for variations in hue. The NCS deals with four separate scales, i.e., there are four hue partitions: from red to blue, from blue to green, from green to yellow, and from yellow to red. Each hue partition of visual content can be illustrated as a bi-polar diagram (modified from Reference [4]), as shown in Figure 1, and as triangular membership functions, as shown in Figure 2. The end point of one scale is identical to the starting point of the next scale. The membership functions are made to mutually overlap.

In Figure 3, partition into four chromatic elementary colors is given directly by the opponent color theory as positions of these colors on the circle. This illustrates graphically the mutual exclusiveness of the attributes of opposite colors. The arc of each quadrant of the circle, which designates all of the imaginary maximal hues of this hue scale, is divided into 100 equal steps. For example, a color that looks equally yellowish and reddish is marked in the middle of the arc between Y and R. According to the NCS notation for hue, this will be Y50R, which means 50 hue steps from Y to R. As another example, the notation R70B indicates a red-blue color situated 70 out of 100 steps from red to blue, i.e., 70% relative bluishness (30% relative reddishness). The markings on the hue circle referred to here do not represent single colors but all imaginable colors of particular hues (i.e., all colors with the same proportion of yellowness-redness and redness-blueness, respectively, for these two examples).

2.2. NCS color triangle. The graphical illustration of all colors of a given hue is shown in the NCS equilateral triangle (as in Hering's original model), where each point represents a certain nuance (see Figure 4). The uni-dimensional "dimension" nuance is thus the location for all colors, of all hues, with a specific relationship between the parameters of whiteness, blackness, and chromaticness indicated arrows.

2.3. NCS Chromaticness. Chromaticness corresponds to the dimensional direction of Munsell's Chroma (in the HVC system), but not to the scaling. Chroma is an open-ended scale, which is defined by color samples. On the other hand, NCS chromaticness has two end points, and the scale between these points is divided into 100 equal steps and defines"resemblance to the reference points." Chromaticness in the color triangle is the"resemblance (in %) to the maximal color," by which is meant the color with the given hue, which is devoid of both whiteness and blackness. In the NCS color triangle, the corner point C in Figure 4 represents this maximal color. See also the highest chroma in Reference [1].

2.4. NCS whiteness and blackness. The other corner points W (white) and S (black) represent, respectively, the imaginable elementary colors white and black according to Hering's phenomenological color model. The variable whiteness is defined as the degree of resemblance to the imaginary elementary white (W), and the blackness is defined as the degree of resemblance to the imaginary elementary black (S). The scaling is an equispacing in 100 steps (see Figure 4).

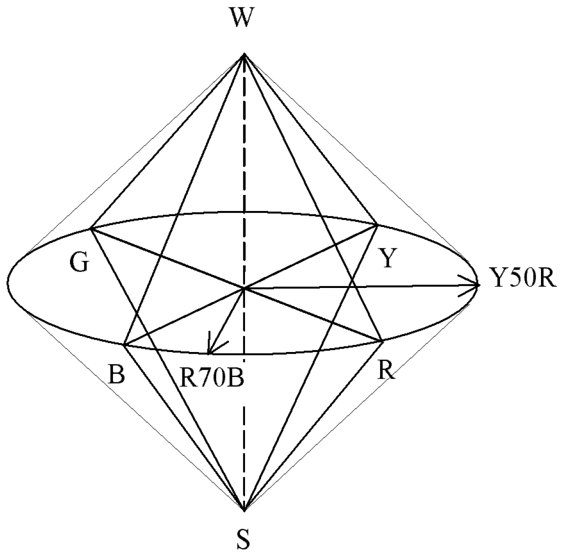

FIGURE 3. NCS color space and color circle

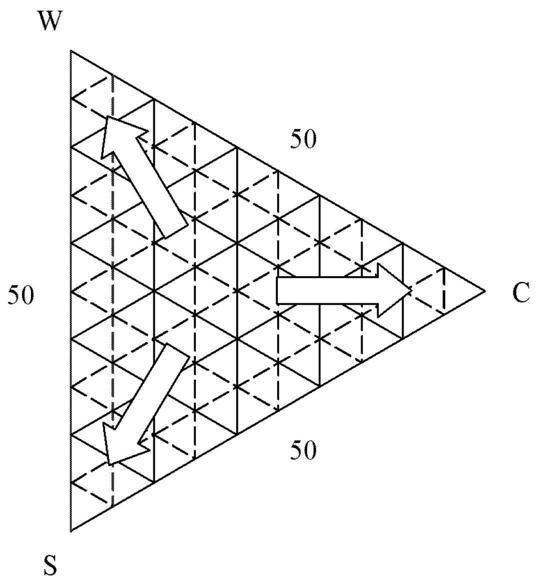

FIGURE 4. NCS color triangle

2.5. **NCS color space.** If we now combine the hue circle with the color triangles for each possible hue, we obtain the three-dimensional NCS color sphere (see Figure 3). Each point within the three-dimensional NCS color sphere represents the perception of a surface color for which the relative resemblances to chromatic elementary colors and to white and black are given by the relative distances to these points in the color sphere [4].

3. **Methods and Results.** Figure 5 illustrates fuzzy input sets of triangular pyramid, fuzzy input, and crisp output on the NCS color triangle, and crisp output on the HLS

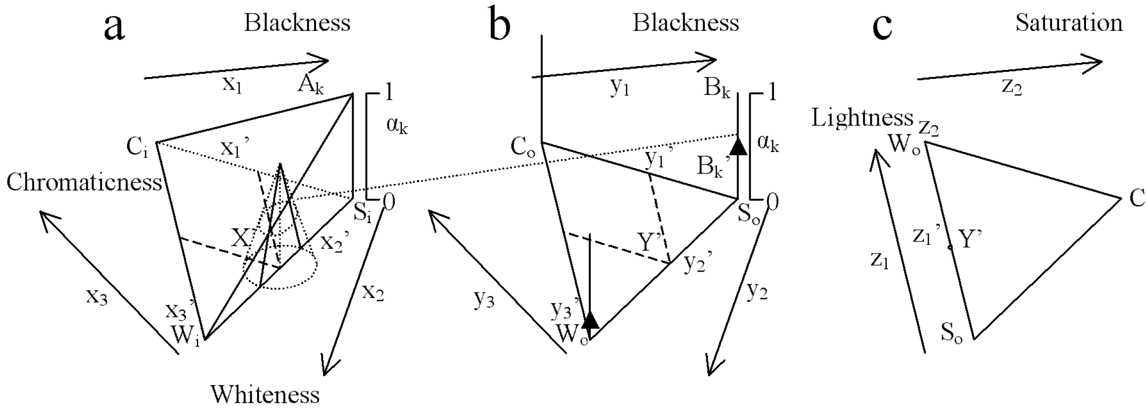

FIGURE 5. Fuzzy system using the membership function of a triangular pyramid on the NCS color triangle, and the HLS tone plane

tone plane. The fuzzy rules are as follows (see also Figure 7):

$$R_1: \text{ IF } X \text{ is } A_1 \text{ THEN } Y \text{ is } B_1 \tag{1}$$

$$R_2: \text{ IF } X \text{ is } A_2 \text{ THEN } Y \text{ is } B_2 \tag{2}$$

$$R_3: \text{ IF } X \text{ is } A_3 \text{ THEN } Y \text{ is } B_3 \tag{3}$$

Rules R_k: if X is A_k, then Y is B_k $(k = 1, 2, 3)$, where k is the rule number, A_k is a fuzzy set of inputs, and B_k is a crisp set of outputs. $X = (x_1, x_2, x_3)$ are input parameters (variable), and $Y = (y_1, y_2, y_3)$ are output parameters. X and Y are fixed on these parameters of NCS. A fuzzy input set A_k has a triangular pyramid form at corner points S_i, W_i, and C_i, and a crisp output set B_k of rule R_k is shown at corner points S_o, W_o, or C_o on the color triangle, and output is B_k if the input is A_k. The fuzzy set B'_k is indicated by the arrow.

The fuzzy inference method is as follows. Let the input be $x_1 = x'_1, x_2 = x'_2$, and $x_3 = x'_3$. *i*: The input of rule R_k, grade $\alpha_k = A_k(X')$, where $k = 1, 2, 3$. *ii*: The output of rule R_k and the α_k level-set are shown as a vertical filled allow. *iii*: $B'_k = \alpha_k B_k$, where B_k is a crisp set, shown in Figure 5b (center). *iv*: The complete inference result B' of rules R_1, R_2, and R_3.

$$B' = \alpha_1 B_1 \cup \alpha_2 B_2 \cup \alpha_3 B_3 = B'_1 \cup B'_2 \cup B'_3 \tag{4}$$

The output parameter, $Y' = (y'_1, y'_2, y'_3)$ corresponds to the coordinate of the central axis of the membership function of B', which is a de-fuzzification. In addition, in Figure 5c (right), $Y' = (z'_1, z'_2)$ corresponds to the coordinate of the HLS system. z'_1 (on the lightness axis) is calculated from y'_1 and y'_2, and z'_2 (on the saturation axis) is equal to y'_3.

Figure 6a (left) illustrates fuzzy inputs $(I_1 - I_6)$ of achromatic colors on the NCS color triangle. The fuzzy inputs are formed by conical membership functions, and the membership functions are made to mutually overlap. The edge of the basal plane (circle) of the conical membership function passes through the centers of the overlapping circles. The fuzzy inputs $(I_{01} - I_{06})$ are formed by a triangular membership function on the $W_i - S_i$ line corresponding to the achromatic colors of the HLS system.

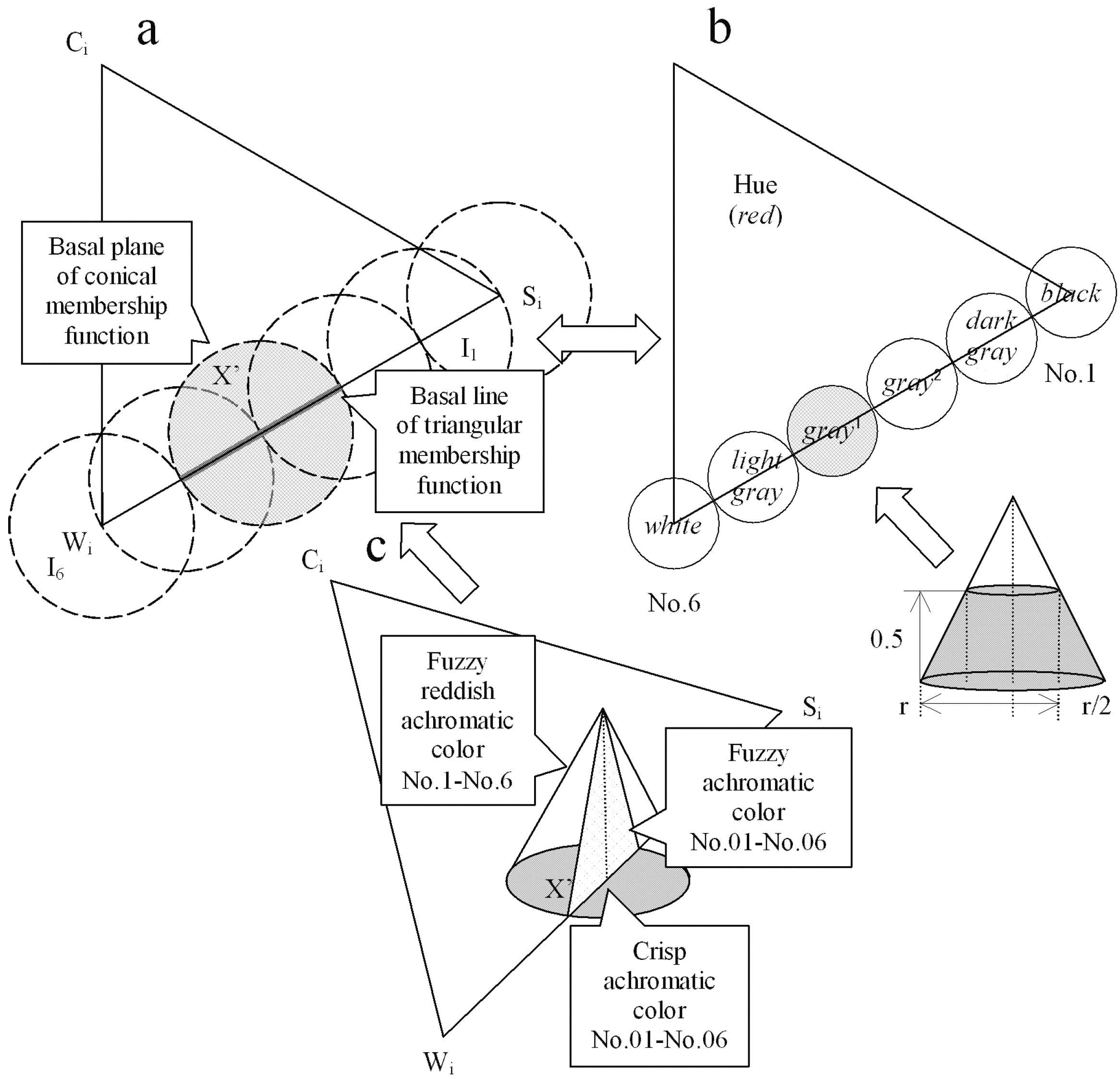

FIGURE 6. Fuzzy inputs of achromatic colors on the NCS color triangle and top areas of 0.5 level-sets with reddish achromatic color names

Figure 6b (right) illustrates the arranged modifiers corresponding to the conical membership functions of Figure 6a, where modifiers are shown by areas enclosed by circles (the top of the 0.5 level-set in Figure 6b, right). The achromatic color names and tone modifiers are the same as those used under JIS [2, 3]. The differences are as follows. Two membership functions are used in constructing 'grays' of achromatic colors, because these modifiers occupy a large region on the plane. In this case, for instance, gray^1 and gray^2 mean lighter gray and darker gray, respectively.

Figure 6c (bottom) illustrates crisp achromatic colors, fuzzy achromatic colors, and fuzzy reddish achromatic colors. Reddish achromatic colors are included as achromatic colors [5-8]. Namely, cones (No.1-No.6) include triangles (No.01-No.06) on the $W_i - S_i$

line. The color names are from the fuzzy color-naming system described in a previous paper [7, 8].

Figure 7 illustrates fuzzy inputs ($I_{01} - I_{06}$) on the NCS color triangle. The fuzzy inputs are formed by triangular membership functions, which are made to mutually overlap. The triangular membership functions are included in the conical membership function (Figure 5a and Figure 6a and c). The triangular membership function Proj (I_{04}) on the unknown axis (not an NCS parameter) is one of six projections of six fuzzy inputs ($I_{01} - I_{06}$) by the rays from the right side, and the triangular membership function Proj (I_{04}) on the blackness axis is one of six projections of these inputs by the rays from the lower part.

The intersection of fuzzy input set A_k with input I_n is $A_k \cap I_n$ (see the shaded area in Figure 7 bottom-right). Grade $\alpha'_k = height(A_k \cap I_n)$. If the input is crisp, then grade α'_k changes to grade α_k. S_o is the new blackness. Proj (B_k) is a projection of the output fuzzy set at the corner point S_o (Figure 5b, center).

In the present study, for vague achromatic color inputs to the NCS, this system can translate input data X of vague color to output data Y of simple color on the NCS color triangle. The fuzzy input (No.04) on the NCS is constructed by the center $X' = (x'_1, x'_2, x'_3) = (40, 60, 0)$ as percentages, and the basal line of the triangle indicating vagueness is $2r = 46\%$ long.

Table 1 shows the fuzzy inference results for six crisp inputs (x'_1, x'_2, x'_3). The grades $(\alpha_1, \alpha_2, \alpha_3)$, inference outputs (y'_1, y'_2, y'_3), and HLS output $Y' = (z'_1, z'_2)$ were calculated. The relationships among inputs, grades, and outputs are proportional. The achromatic color names are No.01: black, No.02: dark gray, No.03: gray^2, No.04: gray^1, No.05: light gray, and No.06: white. These numbers correspond to the suffixes of fuzzy input I.

Table 1. Inference results for crisp inputs of achromatic colors

No.	Crisp input			Grade for crisp input			Inference output			HLS output	
	x'_1	x'_2	x'_3	α_1	α_2	α_3	y'_1	y'_2	y'_3	z'_1	z'_2
01	100	0	0	1.00	0.00	0.00	100.0	0.0	0.0	0.0	0.0
02	80	20	0	0.80	0.20	0.00	80.0	20.0	0.0	20.0	0.0
03	60	40	0	0.60	0.40	0.00	60.0	40.0	0.0	40.0	0.0
04	40	60	0	0.40	0.60	0.00	40.0	60.0	0.0	60.0	0.0
05	20	80	0	0.20	0.80	0.00	20.0	80.0	0.0	80.0	0.0
06	0	100	0	0.00	1.00	0.00	0.0	100.0	0.0	100.0	0.0

Table 2. Inference results for fuzzy inputs of achromatic colors

No.	Center of fuzzy input			Grade for fuzzy input			Inference output			HLS output	
	x'_1	x'_2	x'_3	α'_1	α'_2	α'_3	y'_1	y'_2	y'_3	z'_1	z'_2
01	100	0	0	1.00	0.17	0.00	85.7	14.3	0.0	14.3	0.0
02	80	20	0	0.83	0.33	0.00	71.4	28.6	0.0	28.6	0.0
03	60	40	0	0.67	0.50	0.00	57.1	42.9	0.0	42.9	0.0
04	40	60	0	0.50	0.67	0.00	42.9	57.1	0.0	57.1	0.0
05	20	80	0	0.33	0.83	0.00	28.6	71.4	0.0	71.4	0.0
06	0	100	0	0.17	1.00	0.00	14.3	85.7	0.0	85.7	0.0

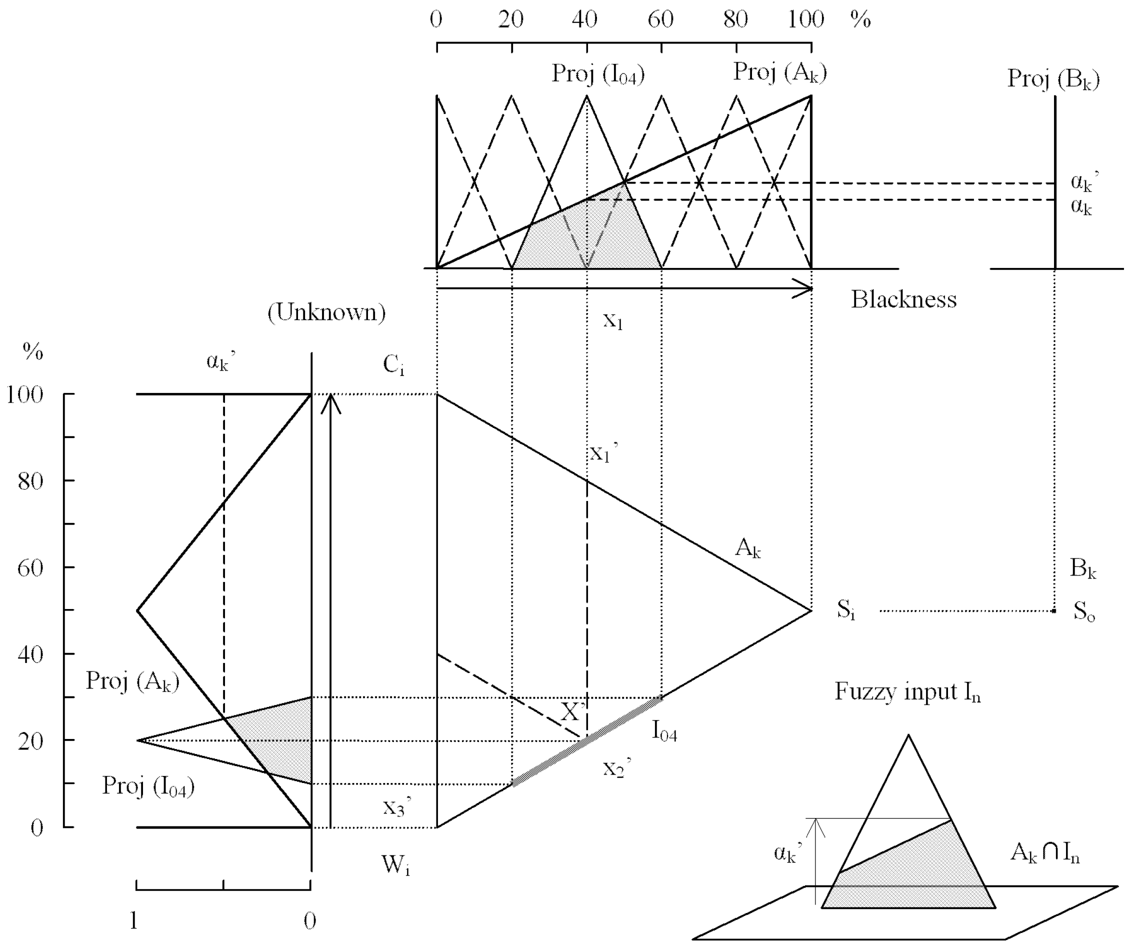

FIGURE 7. Membership functions of triangular pyramid on the NCS color triangle and one of six triangular fuzzy inputs (vague achromatic colors)

Table 2 shows the fuzzy inference results for six fuzzy inputs. The fuzzy inputs on the NCS are constructed by the centers (x_1', x_2', x_3') in the same manner as in Table 1, and the basal line of the triangle-indicated vagueness is 46% of the total length. The inputs and outputs are not proportional, which indicates nonlinear information processing in this fuzzy system.

TABLE 3. Inference results for fuzzy inputs of *reddish* achromatic colors

No.	Center of fuzzy input			Grade for fuzzy input			Inference output			HLS output	
	x_1'	x_2'	x_3'	α_1'	α_2'	α_3'	y_1'	y_2'	y_3'	z_1'	z_2'
1	100	0	0	1.00	0.17	0.17	75.0	12.5	12.5	18.8	12.5
2	80	20	0	0.83	0.33	0.19	61.5	24.6	13.9	31.5	13.9
3	60	40	0	0.67	0.50	0.19	49.2	36.9	13.9	43.8	13.9
4	40	60	0	0.50	0.67	0.19	36.9	49.2	13.9	56.2	13.9
5	20	80	0	0.33	0.83	0.19	24.6	61.5	13.9	68.5	13.9
6	0	100	0	0.17	1.00	0.17	12.5	75.0	12.5	81.3	12.5

Table 3 shows the fuzzy inference results for six fuzzy inputs. The fuzzy inputs on the NCS are constructed by the centers (x_1', x_2', x_3') in the same manner as the crisp inputs in Table 1, and the radius ($r = 23\%$) of the basal plane (circle) of the cone indicated vagueness (see Figure 6). The inputs and outputs are not proportional, which also indicates nonlinear information processing in this fuzzy system.

Figure 8 illustrates the relationship between the lightness z_1 and the saturation z_2 obtained from data (z_1', z_2') in Tables 1, 2, and 3. Filled squares (FC) are outputs for fuzzy inputs of chromatic colors, and filled circles (CC) are outputs for crisp inputs of chromatic colors shown previously in References [9, 11]. In addition, open squares (FA) are outputs for fuzzy inputs of achromatic colors, and open circles (CA) are outputs for crisp inputs of achromatic colors. Circles indicate the corresponding coordinates for the input points in the NCS. Squares indicate the centers of tone triangles. That is, points indicated by circles move toward points indicated by square using fuzzy inference.

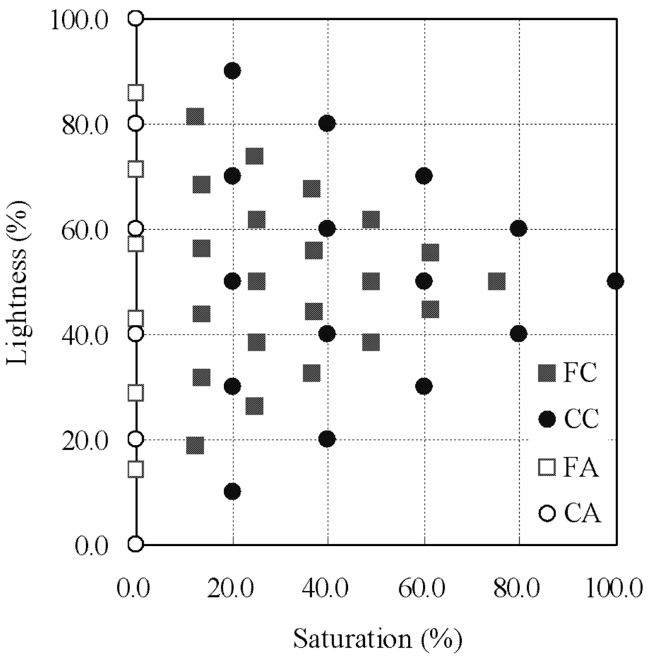

FIGURE 8. Fuzzy inference results for crisp (circle) and fuzzy (square) inputs on the HLS tone plane

Next, the relationships among crisp inputs (centers of large circles), conical fuzzy inputs (large circles), top areas of 0.5 level-sets of each modifier (medium circles), and fuzzy inference outputs (circles and squares) on the NCS color triangle are examined (see Figure 6).

Figure 9 shows a schematic illustration, which focuses on one light grayish area. The center of the modifier "light gray" as the coordinate of conical fuzzy input moves to the area (middle circle) of neighboring "light grayish", because vagueness of the modifier is inferred. The center of the modifier "light gray", as the coordinate of triangular fuzzy input, moves toward the blackness direction. The center of the modifier "white", as

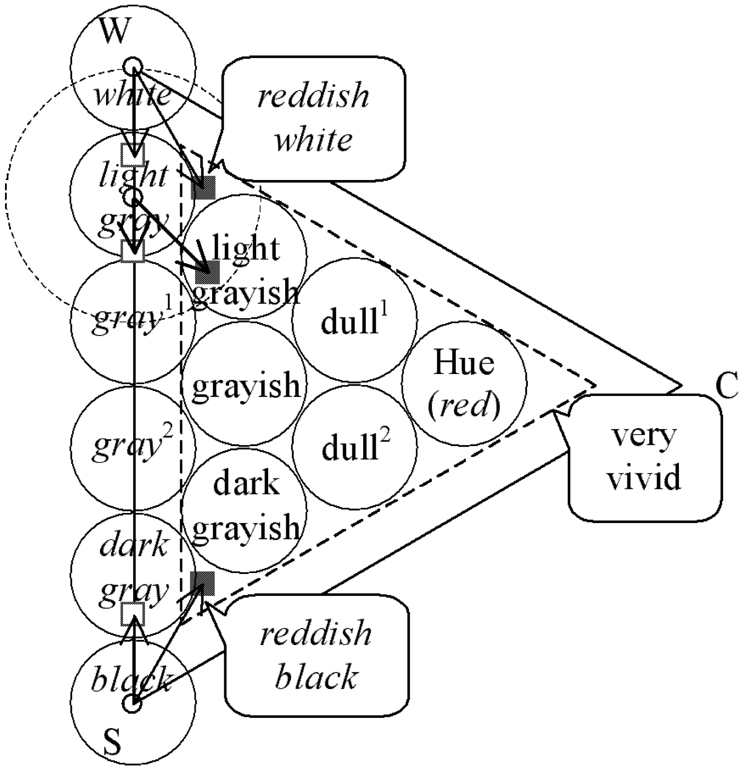

FIGURE 9. Schematic gathering effects of outputs for triangular and conical fuzzy input

the coordinate of conical fuzzy input, moves approximately 75% of the way toward the whiteness direction. The center of the modifier "black", as the coordinate of conical fuzzy input, also moves approximately 75% of the way toward the blackness direction. The center of the modifier "white", as the coordinate of triangular fuzzy input, moves to the area of neighboring "light gray". The center of the modifier "black", as the coordinate of triangular fuzzy input, also moves to the area of neighboring "dark gray" [9-11]. It is thus clarified that modified achromatic colors occur in positions between achromatic colors and chromatic colors, using a small narrow conical membership function. Namely, as the radius r of the basal plane of the cone indicated that vagueness is small ($r < 23\%$), only achromatic colors with vagueness become modified achromatic colors.

4. **Conclusions.** The present paper proposes a fuzzy system that, for a known hue name (e.g., red), can extract crisp outputs of the NCS and HLS. It is difficult to construct such a fuzzy system on the tone plane in the HLS system because the triangular pyramid membership function of the lightness is quite complicated and does not fit on the tone triangle. That is, the triangular pyramid is suited to NCS whiteness and blackness rather than HLS lightness. A limitation of this method is that gravity calculation is not available for only two attributes, lightness (Value) and saturation (Chroma), on their tone plane in other color systems. All three attributes (whiteness, blackness, chromaticness) of the tone system of the NCS are very suitable to apply fuzzy set theoretical methods.

The system also extracts, in a simple manner, the membership grades from the projection of a triangular (or conical) membership function of a vague achromatic color input. Three parameters associated with respective grades indicate vague colors and output the center of gravity as a crisp color value, so the NCS does not have a 'lightness' attribute. This system will help us to ensure important color information (e.g. vagueness, shadings of color) of goods and art by reducing the confusion of achromatic and chromatic colors.

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